Math III Final Review

1. a. $f(-1)=0$

   b. $f(0)=-6$

   c. $f(x)=0;\ x=-2,-1,2,3$

   d. $f(x)=-7;\ x=-2.5,3.5$

   e. NEITHER

   f. Local min: $(\frac{1}{2},-6.5)$ Local max: None

   g. $D:\{x|x\in\mathbb{R}\}$ $R:\{y|y\leq 2\}$

   h. $(-\infty,-1.5)\cup(0.5,2.5)$

   i. $(-1.5,0.5)\cup(2.5,\infty)$

   j. $(-0.5,1.5)$

   k. $(-\infty,-0.5)\cup(1.5,\infty)$

   l. $-2,-1,2,3$

   m. $f(x)=a(x+2)(x+1)(x-2)(x-3)$

   $-6=a\cdot(2)(1)(-2)(-3)$

   $-6=a\cdot 12$

   $a=-\frac{1}{2}$

$f(x)=-\frac{1}{2}(x+2)(x+1)(x-2)(x-3)$

2. $f(x) = \frac{2x-6}{x+4}$

a. $x = \frac{2y-6}{y+4}$

$x(y+4) = 2y-6$

$xy+4x = 2y-6$

$xy-2y = -4x-6$

$y(x-2) = -4x-6$

$f^{-1}(x) = \frac{-4x-6}{x-2}$

b. $f(f^{-1}(x)) = \frac{2\left(\frac{-4x-6}{x-2}\right)-6}{\frac{-4x-6}{x-2}+4}$

$= \frac{\frac{-8x-12}{x-2} - \frac{6x-12}{x-2}}{\frac{-4x-6}{x-2} + \frac{4x-8}{x-2}}$

$= \frac{\frac{-14x}{x-2}}{\frac{-14}{x-2}}$

$= \frac{-14x}{x-2} \cdot \frac{x-2}{-14}$

$= \frac{-14x}{-14}$

$= x$

c. $f(x)$

D: $\{x \mid x \neq -4\}$

R: $\{y \mid y \neq 2\}$

$f^{-1}(x)$

D: $\{x \mid x \neq 2\}$

R: $\{y \mid y \neq -4\}$

2. d. $f(0) = \frac{2(0)-6}{0+4}$

$= \frac{-6}{4}$

$f(0) = -\frac{3}{2}$

e. $\frac{2x-6}{x+4} = 3$

$2x-6 = 3x+12$

$-x = 18$

$x = 18$

f. $f(-x) = \frac{-2x-6}{-x+4}$

$= \frac{-(2x+6)}{-(x-4)}$

$= \frac{2x+6}{x-4}$

NEITHER

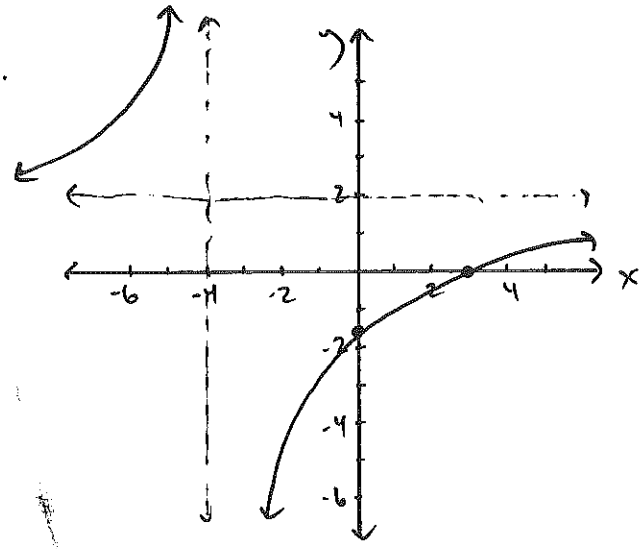
g. V.A: $x = -4$

H.A: $y = 2$

h. x-int: $(3, 0)$

y-int: $(0, -\frac{3}{2})$

i.



$f(-5) = \frac{-10-6}{-1}$

$= 16$

3. SEE PACKET

3. Let $f(x) = |x|$. For each of the following, sketch a graph of the transformation in Figure 4 and write the simplified formula for the function. Describe the order of transformations, being as specific as possible and listing them in an appropriate order.

(a) $y = -f(x)$
(b) $y = f(x+1)$
(c) $y = 2f(x)$
(d) $y = f(x) + 3$
(e) $y = 2f(x+1) + 3$
(f) $y = f(3x)$

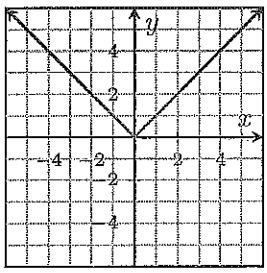


FIGURE 3. Graph of $y = |x|$

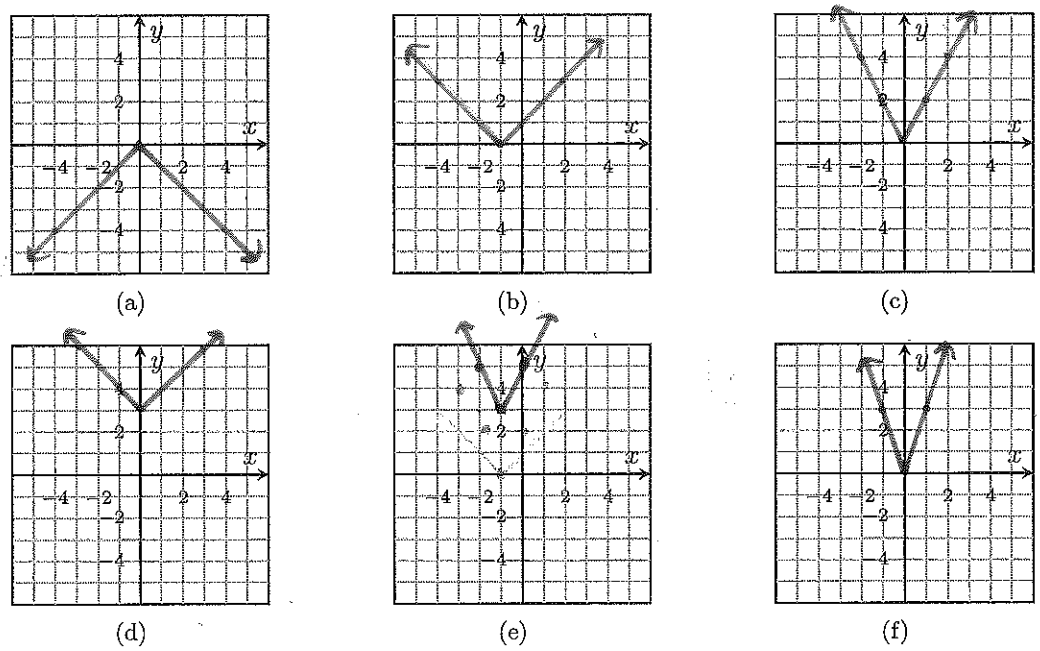


FIGURE 4

4. For each function below, identify the original (or basic) function and explain how the graph is a transformation of the graph of the original function. State all steps to this transformation in an appropriate order.

(a) $g(x) = -\frac{1}{3}(4(x-7))^3 - 2$
(b) $g(x) = 7\ln(x+4) + 5$
(c) $g(x) = \sqrt{-\frac{1}{4}(x+1)} - 3$

5. The point $(-4, 16)$ is on the graph of $y = f(x)$. Determine the point on the graph of...

(a) $y = f(x-5) - 7$
(b) $y = -f(4x)$
(c) $y = 3f(-x)$
(d) $y = -\frac{1}{8}f(2(x+3)) + 5$

4. a. $g(x) = -\frac{1}{3}(4(x-7))^3 - 2$

PARENT: $y = x^3$

1. Reflect across x-axis, and vertically compress by $\frac{1}{3}$.
2. Horizontally compress by $\frac{1}{4}$
3. Shift Right 7
4. Shift down 2

b. $g(x) = 7\ln(x+4) + 5$

PARENT: $y = \ln(x)$

1. Vertical stretch by a factor of 7.
2. Shift left 4
3. Shift up 5.

c. $g(x) = \sqrt{-\frac{1}{4}(x+1)} - 3$

PARENT: $y = \sqrt{x}$

1. Reflect across y-axis and horizontal stretch by 4.
2. Shift left 1
3. Shift down 3.

5. a. (1, 9)

b. (-1, -16)

c. (4, 48)

d. (-5, 3)

6. Complete Table 1 below using the given values in the table. If any value is undefined, write "undefined."

TABLE 1

x	-2	-1	0	1	2
$f(x)$	2	1	0	1	2
$g(x)$	4	2	0	-2	-4
$(g \circ f)(x)$	-4	-2	0	-2	-4
$(g \cdot f)(x)$	8	2	0	-2	-8
$f(x) + g(x)$	6	3	0	-1	-2
$\frac{f(x)}{g(x)}$	$\frac{1}{2}$	$\frac{1}{2}$	und.	$-\frac{1}{2}$	$-\frac{1}{2}$

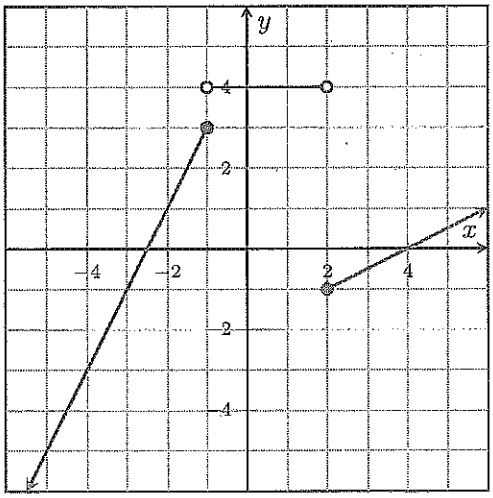
$g(f(x))$

$g(x) \cdot f(x)$

7. Find a formula for the piecewise-defined function graphed in Figure 5 below.

$$f(x) = \begin{cases} 2x+6, & x \le -1 \\ 4, & -1 < x < 2 \\ \frac{1}{2}x - 2, & x \ge 2 \end{cases}$$

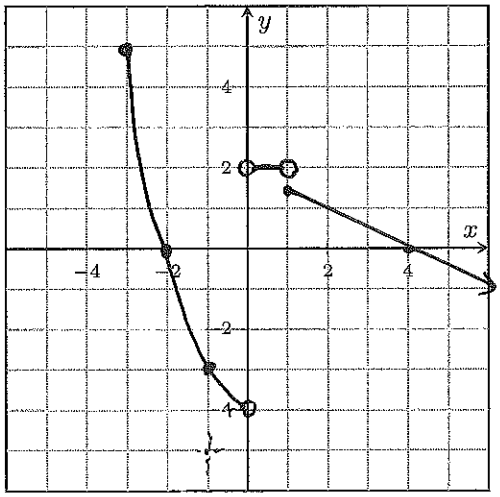
FIGURE 5. Graph of $y = f(x)$



8. In Figure 6, graph the piecewise function defined by

$$f(x) = \begin{cases} x^2 - 4, & -3 \le x < 0 \\ 2, & 0 < x < 1 \\ -\frac{1}{2}x + 2, & x \ge 1 \end{cases}$$

FIGURE 6. Graph of $y = f(x)$



9. The volume, $V(r)$ (in cubic centimeters) of a circular balloon of radius r (in centimeters) is given by $V(r) = \frac{4}{3}\pi r^3$. As someone blows air into the balloon, the radius of the balloon as a function of time t (in seconds) is given by $r = g(t) = 2t$.

(a) Find and interpret $V(3)$.

(b) Find and interpret $g(3)$.

(c) Find and interpret $V(g(3))$.

(d) Find and interpret $V(g(t))$.

(e) Explain why $g(V(r))$ in nonsense.

6. SEE PACKET

7. SEE PACKET

8. SEE PACKET

9. a. $V(3) = \frac{4}{3}\pi(3)^3$

$V(3) = 113.10$

when the radius is 3 cm, the volume of the balloon is 113.10 cu. cm.

b. $g(3) = 2(3)$

$g(3) = 6$

After blowing into the balloon for 3 seconds, the radius would be 6 cm.

c. $V(g(3)) = V(6)$

$= \frac{4}{3}\pi(6)^3$

$= 904.78$

After inflating a balloon for 3 seconds, the volume would be 904.78 cu. cm.

d. $V(g(t)) = \frac{4}{3}\pi(2t)^3$

$= \frac{4}{3}\pi 8t^3$

$= \frac{32}{3}\pi t^3$

$V(g(t))$ gives the volume of a ballon based on how many seconds it's inflated.

e. $g(V(r))$ would give the amount of time a balloon has been inflated for a given Volume ... but you can't know the volume unless you know how long it was inflated.

10. a. $f(g(2)) = f(3(2)^2+1)$

$= f(12+1)$

$= f(13)$

$= \frac{2}{3(13)+1}$

$= \frac{2}{40}$

$f(g(2)) = \frac{1}{20}$

b. $(h \circ f)(1) = h(f(1))$

$= h\left(\frac{2}{3+1}\right)$

$= h\left(\frac{1}{2}\right)$

$= 2\left(\frac{1}{2}\right) - 5$

$(h \circ f)(1) = -4$

c. $(h+g)(1) = 2(1) - 5 + 3(1)^2 + 1$

$= 2 - 5 + 3 + 1$

$(h+g)(1) = 1$

d. $(g \circ g)(0) = g(3(0)^2+1)$

$= g(1)$

$= 3(1)^2 + 1$

$(g \circ g)(0) = 4$

e. $(f-g)(0) = f(0) - g(0)$

$= \frac{2}{1} - 1$

$(f-g)(0) = 1$

f. $(g \cdot g)(x) = (3x^2+1)(3x^2+1)$

$= 9x^4 + 6x^2 + 1$

g. $(f \circ g)(x) = f(g(x))$

$= \frac{2}{3(3x^2+1)+1}$

$= \frac{2}{9x^2+4}$

h. $(g \circ h)(x) = g(h(x))$

$= 3(2x-5)^2 + 1$

$= 3(4x^2 - 20x + 25) + 1$

$= 12x^2 - 60x + 76$

i. $(h \circ h)(x) = 2(2x-5) - 5$

$= 4x - 15$

11. a.

x	-1	0	1	2
y	$\frac{1}{3}$		12	

$\frac{1}{3}b^2 = 12$

$b^2 = 36$

$b = 6$

$y = 2 \cdot 6^x$

b.

x	0	1	2	3	4	5
y			128	$\times b$	$\times b$	$\times b$ 2

$128 \cdot b^3 = 2$

$b^3 = \frac{1}{64}$

$b = \frac{1}{4}$

$y = 2048 \cdot (\frac{1}{4})^x$

12. a. $\log_4(64) = 3$

$4^3 = 64$

b. $\ln(\sqrt{e}) = \frac{1}{2}$

$e^{\frac{1}{2}} = \sqrt{e}$

c. $\log_{10}(\frac{1}{100}) = -2$

$10^{-2} = \frac{1}{100}$

13. a. $7^x - 1 = 4$

$7^x = 5$

$x = \log_7(5)$

$x \approx 0.83$

b. $e^{5x} = 10$

$5x = \ln 10$

$x = \frac{\ln 10}{5}$

$x \approx 0.46$

c. $5e^x = 10$

$e^x = 2$

$x = \ln 2$

$x \approx 0.69$

d. $3^{x^2} = 9^{x+4}$

$3^{x^2} = (3^2)^{x+4}$

$3^{x^2} = 3^{2x+8}$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

e. $3^{2x+1} = 6$

$2x+1 = \log_3(6)$

$x = \frac{\log_3(6) - 1}{2}$

$x \approx 0.32$

13. f. $\log_4(2x+1) = 2$

$2x+1 = 4^2$

$2x+1 = 16$

$x = \frac{15}{2}$

g. $\log_2(x) + \log_2(3) = \log_2(2)$

$\log_2(3x) = \log_2(2)$

$3x = 2$

$x = \frac{2}{3}$

h. $\log_2(x) - \log_2(3) = \log_2(2)$

$\log_2\left(\frac{x}{3}\right) = \log_2(2)$

$\frac{x}{3} = 2$

$x = 6$

i. $2\log_5(x-6) = \log_5(x)$

$\log_5(x-6)^2 = \log_5(x)$

$(x-6)^2 = x$

$x^2 - 12x + 36 = x$

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

$x = 9$, and

$x = 4$ is extraneous

j. $\log_x(\sqrt{3}) = \frac{1}{4}$

$(x^{\frac{1}{4}})^4 = \sqrt{3}^4$

$x = \sqrt{3}^4$

$x = 9$

13. k. $\log(1-x) = 2 + \log(1+x)$

$\log(1-x) - \log(1+x) = 2$

$\log\left(\frac{1-x}{1+x}\right) = 2$

$\frac{1-x}{1+x} = 10^2$

$1-x = 100(1+x)$

$1-x = 100 + 100x$

$-101x = 99$

$x = \frac{-99}{101}$

l. $\log_6(x+4) + \log_6(x+3) = 1$

$\log_6((x+4)(x+3)) = 1$

$x^2 + 7x + 12 = 6$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -6, -1$

The solution set is $\{-1\}$ since $x = -6$ is an extraneous solution.

14. a. $f(750) = 100e^{-0.000124(750)}$

$= 91.12$

91.12% of carbon 14 should remain

b. $f(x) = 70$

$100e^{-0.000124t} = 70$

$e^{-0.000124t} = 0.70$

$-0.000124t = \ln(0.7)$

$t \approx 2876$ years old

15. a. $T(0) = 100e^{-0.1(0)} + 68$

$= 100 \cdot 1 + 68$

$= 168$

The tea was brewed to 168° F.

b. $T(10) = 100e^{-0.1(10)} + 68$

$= 104.79$

After 10 minutes, the tea is at about 105° F.

c. $100e^{-0.1t} + 68 = 80$

$e^{-0.1t} = \frac{12}{100}$

$-0.1t = \ln\left(\frac{12}{100}\right)$

$t \approx 21.20$

The tea will be 80°F after about 21 minutes.

d. $y = 68$

16. a. Tom: \$5000 Jerry: \$4500

b. $T(5) = 5000(1.065)^5$

$= \$6850.43$

$J(t) = 4500\left(1 + \frac{0.065}{12}\right)^{12(5)}$

$= \$6222.68$

c. $4500\left(1 + \frac{0.065}{12}\right)^{12t} = 9000$

$\left(1 + \frac{0.065}{12}\right)^{12t} = 2$

$12t = \log_{\left(1 + \frac{0.065}{12}\right)}(2)$

$t \approx 10.69$ years

d. $5000(1.065)^t = 4500\left(1 + \frac{0.065}{12}\right)^{12t}$

y_1 y_2

(56.96, 180613.26)

After 56.96 years, the accounts will have \$180,613.26

17. a. $f(x) = a(x+1)^2(x-5)$

$-1 = a(0+1)^2(0-5)$

$-1 = a \cdot 1 \cdot -5$

$a = \frac{1}{5}$

$f(x) = \frac{1}{5}(x+1)^2(x-5)$

b. $f(x) = a(x+2)(x+1)x(x-2)(x-5)^3$

$-4 = a(1+2)(1+1)\cdot 1 \cdot (1-2)(1-5)^3$

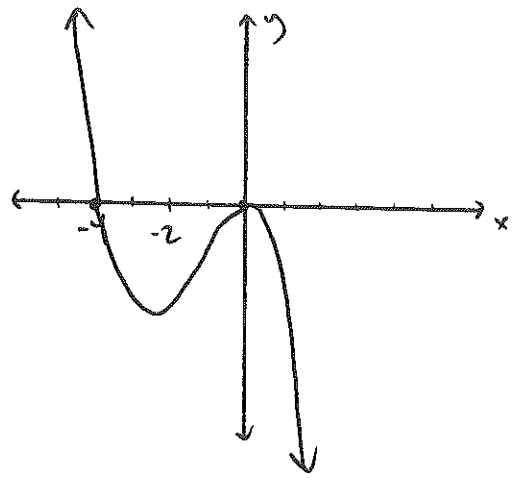
$= a \cdot 3 \cdot 2 \cdot 1 \cdot (-1) \cdot (-64)$

$= a \cdot 384$

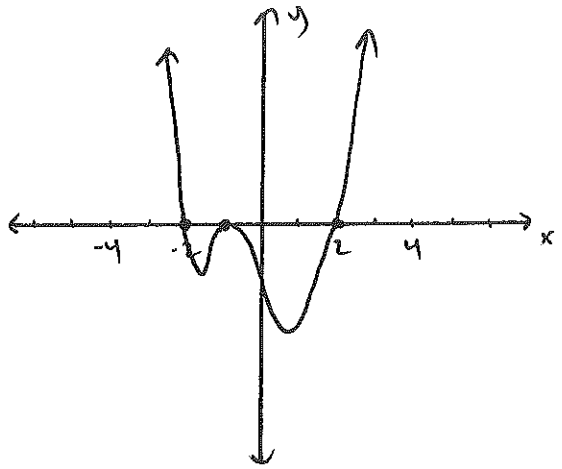
$a = \frac{-4}{384} = -\frac{1}{96}$

$f(x) = -\frac{1}{96}x(x+2)(x+1)(x-2)(x-5)^3$

18. a. $f(x) = -x^2(x+4)$



b. $g(x) = (x-2)(x+1)^2(x+2)$



19. a. $R(x) = \dfrac{2(x+2)}{(x-1)}$

b. $R(x) = \dfrac{x(x-3)}{(x+1)^2(x-4)}$

20. $R(x) = \dfrac{4(x+2)(x-2)^2(x-7)}{(x+3)^2(x-3)(x-6)}$