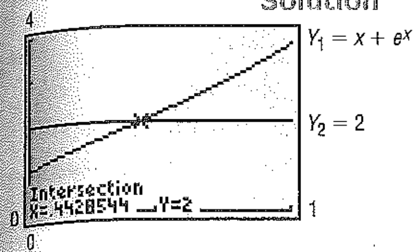
EXAMPLE 7 **Solving Equations Using a Graphing Utility**

Solve: $x + e^x = 2$

Express the solution(s) rounded to two decimal places.

Figure 43



Solution The solution is found by graphing $Y_1 = x + e^x$ and $Y_2 = 2$. Since Y_1 is an increasing function (do you know why?), there is only one point of intersection for Y_1 and Y_2. Figure 43 shows the graphs of Y_1 and Y_2. Using the INTERSECT command reveals that the solution is 0.44, rounded to two decimal places.

Now Work PROBLEM 71

4.6 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Solve $x^2 - 7x - 30 = 0$. (pp. 137–143)
2. Solve $(x + 3)^2 - 4(x + 3) + 3 = 0$. (pp. 144–145)
3. Approximate the solution(s) to $x^3 = x^2 - 5$ using a graphing utility. (pp. B6–B7)
4. Approximate the solution(s) to $x^3 - 2x + 2 = 0$ using a graphing utility. (pp. B6–B7)

Skill Building

In Problems 5–40, solve each logarithmic equation. Express any irrational solution in exact form and as a decimal rounded to three decimal places.

5. $\log_4 x = 2$
6. $\log(x + 6) = 1$
7. $\log_2(5x) = 4$
8. $\log_3(3x - 1) = 2$
9. $\log_4(x + 2) = \log_4 8$
10. $\log_5(2x + 3) = \log_5 3$
11. $\frac{1}{2}\log_3 x = 2\log_3 2$
12. $-2\log_4 x = \log_4 9$
13. $3\log_2 x = -\log_2 27$
14. $2\log_5 x = 3\log_5 4$
15. $3\log_2(x - 1) + \log_2 4 = 5$
16. $2\log_3(x + 4) - \log_3 9 = 2$
17. $\log x + \log(x + 15) = 2$
18. $\log x + \log(x - 21) = 2$
19. $\log(2x + 1) = 1 + \log(x - 2)$
20. $\log(2x) - \log(x - 3) = 1$
21. $\log_2(x + 7) + \log_2(x + 8) = 1$
22. $\log_6(x + 4) + \log_6(x + 3) = 1$
23. $\log_8(x + 6) = 1 - \log_8(x + 4)$
24. $\log_5(x + 3) = 1 - \log_5(x - 1)$
25. $\ln x + \ln(x + 2) = 4$
26. $\ln(x + 1) - \ln x = 2$
27. $\log_3(x + 1) + \log_3(x + 4) = 2$
28. $\log_2(x + 1) + \log_2(x + 7) = 3$
29. $\log_{1/3}(x^2 + x) - \log_{1/3}(x^2 - x) = -1$
30. $\log_4(x^2 - 9) - \log_4(x + 3) = 3$
31. $\log_a(x - 1) - \log_a(x + 6) = \log_a(x - 2) - \log_a(x + 3)$
32. $\log_a x + \log_a(x - 2) = \log_a(x + 4)$
33. $2\log_5(x - 3) - \log_5 8 = \log_5 2$
34. $\log_3(x) - 2\log_3 5 = \log_3(x + 1) - 2\log_3 10$
35. $2\log_6(x + 2) = 3\log_6 2 + \log_6 4$
36. $3(\log_7 x - \log_7 2) = 2\log_7 4$
37. $2\log_{13}(x + 2) = \log_{13}(4x + 7)$
38. $\log(x - 1) = \frac{1}{3}\log 2$
39. $(\log_3 x)^2 - 5(\log_3 x) = 6$
40. $\ln x - 3\sqrt{\ln x} + 2 = 0$

In Problems 41–68, solve each exponential equation. Express any irrational solution in exact form and as a decimal rounded to three decimal places.

41. $2^{x-5} = 8$
42. $5^{-x} = 25$
43. $2^x = 10$
44. $3^x = 14$
45. $8^{-x} = 1.2$
46. $2^{-x} = 1.5$
47. $5(2^{3x}) = 8$
48. $0.3(4^{0.2x}) = 0.2$
49. $3^{1-2x} = 4^x$
50. $2^{x+1} = 5^{1-2x}$
51. $\left(\frac{3}{5}\right)^x = 7^{1-x}$
52. $\left(\frac{4}{3}\right)^{1-x} = 5^x$
53. $1.2^x = (0.5)^{-x}$
54. $0.3^{1+x} = 1.7^{2x-1}$
55. $\pi^{1-x} = e^x$
56. $e^{x+3} = \pi^x$
57. $2^{2x} + 2^x - 12 = 0$
58. $3^{2x} + 3^x - 2 = 0$
59. $3^{2x} + 3^{x+1} - 4 = 0$
60. $2^{2x} + 2^{x+2} - 12 = 0$
61. $16^x + 4^{x+1} - 3 = 0$
62. $9^x - 3^{x+1} + 1 = 0$
63. $25^x - 8 \cdot 5^x = -16$
64. $36^x - 6 \cdot 6^x = -9$
65. $3 \cdot 4^x + 4 \cdot 2^x + 8 = 0$
66. $2 \cdot 49^x + 11 \cdot 7^x + 5 = 0$
67. $4^x - 10 \cdot 4^{-x} = 3$
68. $3^x - 14 \cdot 3^{-x} = 5$

In Problems 69–82, use a graphing utility to solve each equation. Express your answer rounded to two decimal places.

69. $\log_5(x + 1) - \log_4(x - 2) = 1$
70. $\log_2(x - 1) - \log_6(x + 2) = 2$
71. $e^x = -x$
72. $e^{2x} = x + 2$
73. $e^x = x^2$
74. $e^x = x^3$

EXAMPLE 5 **Solving an Exponential Equation**

Solve: $5^{x-2} = 3^{3x+2}$

Solution Because the bases are different, first apply property (7), Section 4.5 (take the natural logarithm of each side), and then use a property of logarithms. The result is an equation in x that can be solved.

$$
\begin{aligned}
5^{x-2} &= 3^{3x+2} \\
\ln 5^{x-2} &= \ln 3^{3x+2} && \text{If } M = N, \ln M = \ln N. \\
(x-2)\ln 5 &= (3x+2)\ln 3 && \ln M^r = r \ln M \\
(\ln 5)x - 2\ln 5 &= (3\ln 3)x + 2\ln 3 && \text{Distribute.} \\
(\ln 5)x - (3\ln 3)x &= 2\ln 3 + 2\ln 5 && \text{Place terms involving } x \text{ on the left.} \\
(\ln 5 - 3\ln 3)x &= 2(\ln 3 + \ln 5) && \text{Factor.} \\
x &= \frac{2(\ln 3 + \ln 5)}{\ln 5 - 3\ln 3} && \text{Exact solution} \\
&\approx -3.212 && \text{Approximate solution}
\end{aligned}
$$

The solution set is $\left\{\dfrac{2(\ln 3 + \ln 5)}{\ln 5 - 3\ln 3}\right\}$.

NOTE: Because of the properties of logarithms, exact solutions involving logarithms often can be expressed in multiple ways. For example, the solution to $5^{x-2} = 3^{3x+2}$ from Example 5 can be expressed equivalently as $\dfrac{2\ln 15}{\ln 5 - \ln 27}$ or $\dfrac{\ln 225}{\ln(5/27)}$ among others. Do you see why?

Now Work PROBLEM 53

The next example deals with an exponential equation that is quadratic in form.

EXAMPLE 6 **Solving an Exponential Equation That Is Quadratic in Form**

Solve: $4^x - 2^x - 12 = 0$

Solution Note that $4^x = (2^2)^x = 2^{(2x)} = (2^x)^2$, so the equation is quadratic in form and can be written as

$$(2^x)^2 - 2^x - 12 = 0 \qquad \text{Let } u = 2^x; \text{ then } u^2 - u - 12 = 0.$$

Now factor as usual.

$$
\begin{aligned}
(2^x - 4)(2^x + 3) &= 0 && (u-4)(u+3) = 0 \\
2^x - 4 = 0 \quad \text{or} \quad 2^x + 3 &= 0 && u - 4 = 0 \quad \text{or} \quad u + 3 = 0 \\
2^x = 4 \qquad\qquad 2^x &= -3 && u = 2^x = 4 \qquad u = 2^x = -3
\end{aligned}
$$

The equation on the left has the solution $x = 2$, since $2^x = 4 = 2^2$; the equation on the right has no solution, since $2^x > 0$ for all x. The only solution is 2. The solution set is $\{2\}$.

Now Work PROBLEM 61

3 Solve Logarithmic and Exponential Equations Using a Graphing Utility

The algebraic techniques introduced in this section to obtain exact solutions apply only to certain types of logarithmic and exponential equations. Solutions for other types are usually studied in calculus, using numerical methods. For such types, a graphing utility can be used to approximate the solution.

75. $\ln x = -x$
76. $\ln(2x) = -x + 2$
77. $\ln x = x^3 - 1$
78. $\ln x = -x^2$
79. $e^x + \ln x = 4$
80. $e^x - \ln x = 4$
81. $e^{-x} = \ln x$
82. $e^{-x} = -\ln x$

Mixed Practice

In Problems 83–96, solve each equation. Express irrational solutions in exact form and as a decimal rounded to three decimal places.

83. $\log_9(7x - 5) = \log_3(x + 1)$
[**Hint:** Change $\log_9(7x - 5)$ to base 3.]

84. $\log_2(x + 1) - \log_4 x = 1$

85. $\log_2(3x + 2) - \log_4 x = 3$

86. $\log_{16} x + \log_4 x + \log_2 x = 7$

87. $\log_9 x + 3\log_3 x = 14$

88. $2(\log_4 x)^2 + 3\log_8 x = \log_2 16$

89. $(\sqrt[3]{2})^{2-x} = 2^{x^2}$

90. $\log_2 x^{\log_2 x} = 4$

91. $\dfrac{e^x + e^{-x}}{2} = 1$
[**Hint:** Multiply each side by e^x.]

92. $\dfrac{e^x + e^{-x}}{2} = 3$

93. $\dfrac{e^x - e^{-x}}{2} = 2$

94. $\dfrac{e^x - e^{-x}}{2} = -2$

95. $\log_5 x + \log_3 x = 1$
[**Hint:** Use the Change-of-Base Formula.]

96. $\log_2 x + \log_6 x = 3$

97. $f(x) = \log_2(x + 3)$ and $g(x) = \log_2(3x + 1)$.
(a) Solve $f(x) = 3$. What point is on the graph of f
(b) Solve $g(x) = 4$. What point is on the graph of g
(c) Solve $f(x) = g(x)$. Do the graphs of f and g intersect? If so, where?
(d) Solve $(f + g)(x) = 7$.
(e) Solve $(f - g)(x) = 2$.

98. $f(x) = \log_3(x + 5)$ and $g(x) = \log_3(x - 1)$.
(a) Solve $f(x) = 2$. What point is on the graph of f?
(b) Solve $g(x) = 3$. What point is on the graph of g?
(c) Solve $f(x) = g(x)$. Do the graphs of f and g intersect? If so, where?
(d) Solve $(f + g)(x) = 3$.
(e) Solve $(f - g)(x) = 2$.

99. (a) Graph $f(x) = 3^{x+1}$ and $g(x) = 2^{x+2}$, on the same Cartesian plane.
(b) Find the point(s) of intersection of the graphs of f and g by solving $f(x) = g(x)$. Round answers to three decimal places. Label any intersection points on the graph drawn in part (a).
(c) Based on the graph, solve $f(x) > g(x)$.

100. (a) Graph $f(x) = 5^{x-1}$ and $g(x) = 2^{x+1}$, on the same Cartesian plane.
(b) Find the point(s) of intersection of the graphs of f and g by solving $f(x) = g(x)$. Label any intersection points on the graph drawn in part (a).
(c) Based on the graph, solve $f(x) > g(x)$.

101. (a) Graph $f(x) = 3^x$ and $g(x) = 10$ on the same Cartesian plane.
(b) Shade the region bounded by the y-axis, $f(x) = 3^x$, and $g(x) = 10$ on the graph drawn in part (a).
(c) Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

102. (a) Graph $f(x) = 2^x$ and $g(x) = 12$ on the same Cartesian plane.
(b) Shade the region bounded by the y-axis, $f(x) = 2^x$, and $g(x) = 12$ on the graph drawn in part (a).
(c) Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

103. (a) Graph $f(x) = 2^{x+1}$ and $g(x) = 2^{-x+2}$ on the same Cartesian plane.
(b) Shade the region bounded by the y-axis, $f(x) = 2^{x+1}$, and $g(x) = 2^{-x+2}$ on the graph drawn in part (a).
(c) Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

104. (a) Graph $f(x) = 3^{-x+1}$ and $g(x) = 3^{x-2}$ on the same Cartesian plane.
(b) Shade the region bounded by the y-axis, $f(x) = 3^{-x+1}$, and $g(x) = 3^{x-2}$ on the graph draw in part (a).
(c) Solve $f(x) = g(x)$ and label the point of intersection on the graph drawn in part (a).

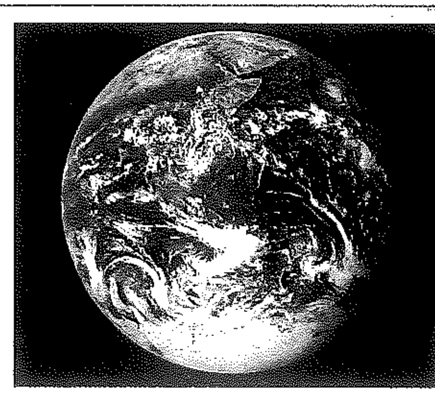
105. (a) Graph $f(x) = 2^x - 4$.
(b) Find the zero of f.
(c) Based on the graph, solve $f(x) < 0$.

106. (a) Graph $g(x) = 3^x - 9$.
(b) Find the zero of g.
(c) Based on the graph, solve $g(x) > 0$.

Applications and Extensions

107. A Population Model The resident population of the United States in 2013 was 316 million people and was growing at a rate of 0.8% per year. Assuming that this growth rate continues, the model $P(t) = 316(1.008)^{t-2013}$ represents the population P (in millions of people) in year t.
(a) According to this model, when will the population of the United States be 419 million people?
(b) According to this model, when will the population of the United States be 490 million people?

Source: U.S. Census Bureau



108. A Population Model The population of the world in 2013 was 7.13 billion people and was growing at a rate of 1.1% per year. Assuming that this growth rate continues, the model $P(t) = 7.13(1.011)^{t-2013}$ represents the population P (in billions of people) in year t.

(a) According to this model, when will the population of the world be 10 billion people?

(b) According to this model, when will the population of the world be 12.2 billion people?

Source: U.S. Census Bureau

109. Depreciation The value V of a Chevy Cruze LT that is t years old can be modeled by $V(t) = 18{,}700(0.84)^t$.

(a) According to the model, when will the car be worth $9000?

(b) According to the model, when will the car be worth $6000?

(c) According to the model, when will the car be worth $2000?

Source: Kelley Blue Book

110. Depreciation The value V of a Honda Civic LX that is t years old can be modeled by $V(t) = 18{,}955(0.905)^t$.

(a) According to the model, when will the car be worth $16,000?

(b) According to the model, when will the car be worth $10,000?

(c) According to the model, when will the car be worth $7500?

Source: Kelley Blue Book

Discussion and Writing

111. Fill in a reason for each step in the following two solutions.

Solve: $\log_3(x-1)^2 = 2$

Solution A	Solution B
$\log_3(x-1)^2 = 2$	$\log_3(x-1)^2 = 2$
$(x-1)^2 = 3^2 = 9$ ______	$2\log_3(x-1) = 2$ ______
$(x-1) = \pm 3$ ______	$\log_3(x-1) = 1$ ______
$x-1 = -3$ or $x-1 = 3$ ______	$x-1 = 3^1 = 3$ ______
$x = -2$ or $x = 4$ ______	$x = 4$ ______

Both solutions given in Solution A check. Explain what caused the solution $x = -2$ to be lost in Solution B.

Retain Your Knowledge

Problems 112–115 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

112. Solve: $4x^3 + 3x^2 - 25x + 6 = 0$.

113. Find the domain of $f(x) = \sqrt{x+3} + \sqrt{x-1}$.

114. For $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{x+5}{x-3}$, find $f \circ g$. Then find the domain of $f \circ g$.

115. Determine whether the function $\{(0,-4), (2,-2), (4,0), (6,2)\}$ is one-to-one.

'Are You Prepared?' Answers

1. $\{-3, 10\}$ **2.** $\{-2, 0\}$ **3.** $\{-1.43\}$ **4.** $\{-1.77\}$

4.7 Financial Models

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Simple Interest (Appendix A, Section A.9, pp. A73–A74)

Now Work the 'Are You Prepared?' problems on page 349.

OBJECTIVES
1. Determine the Future Value of a Lump Sum of Money (p. 340)
2. Calculate Effective Rates of Return (p. 343)
3. Determine the Present Value of a Lump Sum of Money (p. 344)
4. Determine the Rate of Interest or the Time Required to Double a Lump Sum of Money (p. 345)