4.7 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the page listed in red.*

1. What is the interest due if \$500 is borrowed for 6 months at a simple interest rate of 6% per annum? (pp. A73–A74)
2. If you borrow \$5000 and, after 9 months, pay off the loan in the amount of \$5500, what per annum rate of interest was charged? (pp. A73–A74)

Concepts and Vocabulary

3. The total amount borrowed (whether by an individual from a bank in the form of a loan or by a bank from an individual in the form of a savings account) is called the _____.
4. If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is _____ = _____. Interest charged according to this formula is called _____ _____.
5. In working problems involving interest, if the payment period of the interest is quarterly, then interest is paid _____ times per year.
6. The _____ _____ _____ _____ is the equivalent annual simple interest rate that would yield the same amount as compounding n times per year, or continuously, after 1 year.

Skill Building

In Problems 7–14, find the amount that results from each investment.

7. \$100 invested at 4% compounded quarterly after a period of 2 years
8. \$50 invested at 6% compounded monthly after a period of 3 years
9. \$500 invested at 8% compounded quarterly after a period of $2\frac{1}{2}$ years
10. \$300 invested at 12% compounded monthly after a period of $1\frac{1}{2}$ years
11. \$600 invested at 5% compounded daily after a period of 3 years
12. \$700 invested at 6% compounded daily after a period of 2 years
13. \$1000 invested at 11% compounded continuously after a period of 2 years
14. \$400 invested at 7% compounded continuously after a period of 3 years

In Problems 15–22, find the principal needed now to get each amount; that is, find the present value.

15. To get \$100 after 2 years at 6% compounded monthly
16. To get \$75 after 3 years at 8% compounded quarterly?
17. To get \$1000 after $2\frac{1}{2}$ years at 6% compounded daily
18. To get \$800 after $3\frac{1}{2}$ years at 7% compounded monthly
19. To get \$600 after 2 years at 4% compounded quarterly
20. To get \$300 after 4 years at 3% compounded daily
21. To get \$80 after $3\frac{1}{4}$ years at 9% compounded continuously
22. To get \$800 after $2\frac{1}{2}$ years at 8% compounded continuously

In Problems 23–26, find the effective rate of interest.

23. For 5% compounded quarterly
24. For 6% compounded monthly
25. For 5% compounded continuously
26. For 6% compounded continuously

In Problems 27–30, determine the rate that represents the better deal.

27. 6% compounded quarterly or $6\frac{1}{4}$% compounded annually
28. 9% compounded quarterly or $9\frac{1}{4}$% compounded annually
29. 9% compounded monthly or 8.8% compounded daily
30. 8% compounded semiannually or 7.9% compounded daily
31. What rate of interest compounded annually is required to double an investment in 3 years?
32. What rate of interest compounded annually is required to double an investment in 6 years?
33. What rate of interest compounded annually is required to triple an investment in 5 years?
34. What rate of interest compounded annually is required to triple an investment in 10 years?
35. (a) How long does it take for an investment to double in value if it is invested at 8% compounded monthly?
    (b) How long does it take if the interest is compounded continuously?
36. (a) How long does it take for an investment to triple in value if it is invested at 6% compounded monthly?
    (b) How long does it take if the interest is compounded continuously?
37. What rate of interest compounded quarterly will yield an effective interest rate of 7%?
38. What rate of interest compounded continuously will yield an effective interest rate of 6%?

Applications and Extensions

39. **Time Required to Reach a Goal** If Tanisha has \$100 to invest at 8% per annum compounded monthly, how long will it be before she has \$150? If the compounding is continuous, how long will it be?

40. **Time Required to Reach a Goal** If Angela has \$100 to invest at 10% per annum compounded monthly, how long will it be before she has \$175? If the compounding is continuous, how long will it be?

41. **Time Required to Reach a Goal** How many years will it take for an initial investment of \$10,000 to grow to \$25,000? Assume a rate of interest of 6% compounded continuously.

42. **Time Required to Reach a Goal** How many years will it take for an initial investment of \$25,000 to grow to \$80,000? Assume a rate of interest of 7% compounded continuously.

43. **Price Appreciation of Homes** What will a \$90,000 condominium cost 5 years from now if the price appreciation for condos over that period averages 3% compounded annually?

44. **Credit Card Interest** A department store charges 1.25% per month on the unpaid balance for customers with charge accounts (interest is compounded monthly). A customer charges \$200 and does not pay her bill for 6 months. What is the bill at that time?

45. **Saving for a Car** Jerome will be buying a used car for \$15,000 in 3 years. How much money should he ask his parents for now so that, if he invests it at 5% compounded continuously, he will have enough to buy the car?

46. **Paying off a Loan** John requires \$3000 in 6 months to pay off a loan that has no prepayment privileges. If he has the \$3000 now, how much of it should he save in an account paying 3% compounded monthly so that in 6 months he will have exactly \$3000?

47. **Return on a Stock** George contemplates the purchase of 100 shares of a stock selling for \$15 per share. The stock pays no dividends. The history of the stock indicates that it should grow at an annual rate of 15% per year. How much should the 100 shares of stock be worth in 5 years?

48. **Return on an Investment** A business purchased for \$650,000 in 2005 is sold in 2008 for \$850,000. What is the annual rate of return for this investment?

49. **Comparing Savings Plans** Jim places \$1000 in a bank account that pays 5.6% compounded continuously. After 1 year, will he have enough money to buy a computer system that costs \$1060? If another bank will pay Jim 5.9% compounded monthly, is this a better deal?

50. **Savings Plans** On January 1, Kim places \$1000 in a certificate of deposit that pays 6.8% compounded continuously and matures in 3 months. Then Kim places the \$1000 and the interest in a passbook account that pays 5.25% compounded monthly. How much does Kim have in the passbook account on May 1?

51. **Comparing IRA Investments** Will invests \$2000 in his IRA in a bond trust that pays 9% interest compounded semiannually. His friend Henry invests \$2000 in his IRA in a certificate of deposit that pays $8\frac{1}{2}\%$ compounded continuously. Who has more money after 20 years, Will or Henry?

52. **Comparing Two Alternatives** Suppose that April has access to an investment that will pay 10% interest compounded continuously. Which is better: to be given \$1000 now so that she can take advantage of this investment opportunity or to be given \$1325 after 3 years?

53. **College Costs** The average annual cost of college at 4-year private colleges was \$29,056 in the 2012–2013 academic year. This was a 4.2% increase from the previous year.
Source: The College Board
(a) If the cost of college increases by 4.2% each year, what will be the average cost of college at a 4-year private college for the 2030–2031 academic year?
(b) College savings plans, such as a 529 plan, allow individuals to put money aside now to help pay for college later. If one such plan offers a rate of 4% compounded continuously, how much should be put in a college savings plan in 2015 to pay for 1 year of the cost of college at a 4-year private college for an incoming freshman in 2030?

54. **Analyzing Interest Rates on a Mortgage** Colleen and Bill have just purchased a house for \$650,000, with the seller holding a second mortgage of \$100,000. They promise to pay the seller \$100,000 plus all accrued interest 5 years from now. The seller offers them three interest options on the second mortgage:
(a) Simple interest at 12% per annum
(b) $11\frac{1}{2}\%$ interest compounded monthly
(c) $11\frac{1}{4}\%$ interest compounded continuously
Which option is best? That is, which results in paying the least interest on the loan?

55. **2009 Federal Stimulus Package** In February 2009, President Obama signed into law a \$787 billion federal stimulus package. At that time, 20-year Series EE bonds had a fixed rate of 1.3% compounded semiannually. If the federal government financed the stimulus through EE bonds, how much would it have to pay back in 2029? How much interest was paid to finance the stimulus?
Source: U.S. Treasury Department

56. **Per Capita Federal Debt** In 2013, the federal debt was about \$17 trillion. In 2013, the U.S. population was about 316 million. Assuming that the federal debt is increasing about 8.6% per year and the U.S. population is increasing about 0.8% per year, determine the per capita debt (total debt divided by population) in 2020 rounded to the nearest dollar.

Inflation: Problems 57–62 require the following discussion. ***Inflation*** *is a term used to describe the erosion of the purchasing power of money. For example, if the annual inflation rate is 3%, then \$1000 worth of purchasing power now will have only \$970 worth of purchasing power in 1 year because 3% of the original \$1000 (0.03 × 1000 = 30) has been eroded due to inflation. In general, if the rate of inflation averages r per annum over n years, the amount A that \$P will purchase after n years is*

$$A = P \cdot (1 - r)^n$$

where r is expressed as a decimal.

57. **Inflation** If the inflation rate averages 3%, how much will $1000 purchase in 2 years?

58. **Inflation** If the inflation rate averages 2%, how much will $1000 purchase in 3 years?

59. **Inflation** If the amount that $1000 will purchase is only $950 after 2 years, what was the average inflation rate?

60. **Inflation** If the amount that $1000 will purchase is only $930 after 2 years, what was the average inflation rate?

61. **Inflation** If the average inflation rate is 2%, how long is it until purchasing power is cut in half?

62. **Inflation** If the average inflation rate is 4%, how long is it until purchasing power is cut in half?

Problems 63–66 involve zero-coupon bonds. A ***zero-coupon bond*** *is a bond that is sold now at a discount and will pay its face value at the time when it matures; no interest payments are made.*

63. **Zero-Coupon Bonds** A zero-coupon bond can be redeemed in 20 years for $10,000. How much should you be willing to pay for it now if you want a return of:
    (a) 10% compounded monthly?
    (b) 10% compounded continuously?

64. **Zero-Coupon Bonds** A child's grandparents are considering buying a $40,000 face-value, zero-coupon bond at her birth so that she will have money for her college education 17 years later. If they want a rate of return of 8% compounded annually, what should they pay for the bond?

65. **Zero-Coupon Bonds** How much should a $10,000 face-value, zero-coupon bond, maturing in 10 years, be sold for now if its rate of return is to be 8% compounded annually?

66. **Zero-Coupon Bonds** If Pat pays $12,485.52 for a $25,000 face-value, zero-coupon bond that matures in 8 years, what is his annual rate of return?

67. **Time to Double or Triple an Investment** The formula

$$t = \frac{\ln m}{n \ln\left(1 + \frac{r}{n}\right)}$$

can be used to find the number of years t required to multiply an investment m times when r is the per annum interest rate compounded n times a year.
    (a) How many years will it take to double the value of an IRA that compounds annually at the rate of 12%?
    (b) How many years will it take to triple the value of a savings account that compounds quarterly at an annual rate of 6%?
    (c) Give a derivation of this formula.

68. **Time to Reach an Investment Goal** The formula

$$t = \frac{\ln A - \ln P}{r}$$

can be used to find the number of years t required for an investment P to grow to a value A when compounded continuously at an annual rate r.
    (a) How long will it take to increase an initial investment of $1000 to $8000 at an annual rate of 10%?
    (b) What annual rate is required to increase the value of a $2000 IRA to $30,000 in 35 years?
    (c) Give a derivation of this formula.

Problems 69–72 require the following discussion. The ***Consumer Price Index (CPI)*** *indicates the relative change in price over time for a fixed basket of goods and services. It is a cost-of-living index that helps measure the effect of inflation on the cost of goods and services. The CPI uses the base period 1982–1984 for comparison (the CPI for this period is 100). The CPI for January 2013 was 230.28. This means that $100 in the period 1982–1984 had the same purchasing power as $230.28 in January 2013. In general, if the rate of inflation averages r percent per annum over n years, then the CPI index after n years is*

$$\text{CPI} = \text{CPI}_0\left(1 + \frac{r}{100}\right)^n$$

where CPI_0 is the CPI index at the beginning of the n-year period.
Source: U.S. Bureau of Labor Statistics

69. **Consumer Price Index**
    (a) The CPI was 179.9 for 2002 and 229.6 for 2012. Assuming that annual inflation remained constant for this time period, determine the average annual inflation rate.
    (b) Using the inflation rate from part (a), in what year will the CPI reach 300?

70. **Consumer Price Index** If the current CPI is 234.2 and the average annual inflation rate is 2.8%, what will be the CPI in 5 years?

71. **Consumer Price Index** If the average annual inflation rate is 3.1%, how long will it take for the CPI index to double? (A doubling of the CPI index means purchasing power is cut in half.)

72. **Consumer Price Index** The base period for the CPI changed in 1998. Under the previous weight and item structure, the CPI for 1995 was 456.5. If the average annual inflation rate was 5.57%, what year was used as the base period for the CPI?

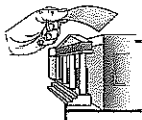
Discussion and Writing

73. Explain in your own words what the term *compound interest* means. What does *continuous compounding* mean?

74. Explain in your own words the meaning of *present value*.

75. **Critical Thinking** You have just contracted to buy a house and will seek financing in the amount of $100,000. You go to several banks. Bank 1 will lend you $100,000 at the rate of 8.75% amortized over 30 years with a loan origination fee of 1.75%. Bank 2 will lend you $100,000 at the rate of 8.375% amortized over 15 years with a loan origination fee of 1.5%. Bank 3 will lend you $100,000 at the rate of 9.125% amortized over 30 years with no loan origination fee. Bank 4 will lend you $100,000 at the rate of 8.625% amortized over 15 years with no loan origination

fee. Which loan would you take? Why? Be sure to have sound reasons for your choice. Use the information in the table to assist you. If the amount of the monthly payment did not matter to you, which loan would you take? Again, have sound reasons for your choice. Compare your final decision with others in the class. Discuss.



	Monthly Payment	Loan Origination Fee
Bank 1	$786.70	$1,750.00
Bank 2	$977.42	$1,500.00
Bank 3	$813.63	$0.00
Bank 4	$992.08	$0.00

Retain Your Knowledge

Problems 76–79 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

76. Find the remainder R when $f(x) = 6x^3 + 3x^2 + 2x - 11$ is divided by $g(x) = x - 1$. Is g a factor of f?

77. Find the real zeros of

$$f(x) = x^5 - x^4 - 15x^3 - 21x^2 - 16x - 20.$$

Then write f in factored form.

78. The function $f(x) = \dfrac{x}{x-2}$ is one-to-one. Find f^{-1}.

79. Solve: $\log_2(x + 3) = 2\log_2(x - 3)$

'Are You Prepared?' Answers

1. \$15 **2.** $13\frac{1}{3}\%$

4.8 Exponential Growth and Decay Models; Newton's Law; Logistic Growth and Decay Models

OBJECTIVES
1 Find Equations of Populations That Obey the Law of Uninhibited Growth (p. 349)
2 Find Equations of Populations That Obey the Law of Decay (p. 351)
3 Use Newton's Law of Cooling (p. 352)
4 Use Logistic Models (p. 354)

1 Find Equations of Populations That Obey the Law of Uninhibited Growth

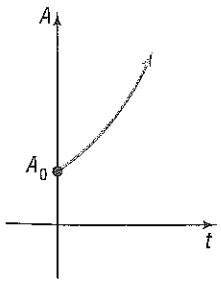
Many natural phenomena have been found to follow the law that an amount A varies with time t according to the function

$$A(t) = A_0 e^{kt} \tag{1}$$

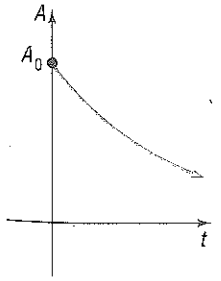
Here A_0 is the original amount $(t = 0)$ and $k \neq 0$ is a constant.

If $k > 0$, then equation (1) states that the amount A is increasing over time; if $k < 0$, the amount A is decreasing over time. In either case, when an amount A varies over time according to equation (1), it is said to follow the **exponential law,** or the **law of uninhibited growth** $(k > 0)$ **or decay** $(k < 0)$. See Figure 44.

Figure 44



(a) $A(t) = A_0 e^{kt}$, $k > 0$



(b) $A(t) = A_0 e^{kt}$, $k < 0$

For example, as seen in Section 4.7, continuously compounded interest was shown to follow the law of uninhibited growth. In this section, additional phenomena that follow the exponential law will be studied.

Cell division is the growth process of many living organisms, such as amoebas, plants, and human skin cells. Based on an ideal situation in which no cells die and no by-products are produced, the number of cells present at a given time follows the law of uninhibited growth. Actually, however, after enough time has passed, growth at an exponential rate will cease due to the influence of factors such as lack of living space and dwindling food supply. The law of uninhibited growth accurately models only the early stages of the cell division process.

The cell division process begins with a culture containing N_0 cells. Each cell in the culture grows for a certain period of time and then divides into two identical