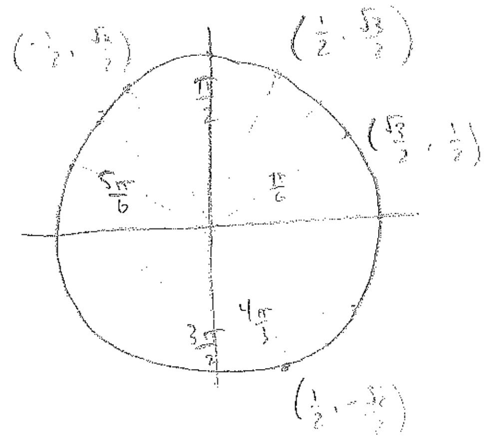
Math 112 Name: Stott - Key

MTH 112 Exam 1 Review
Non-Calculator Portion

6.1 **1.** Fill in the table.

x	arcsin(x)	arccos(x	arctan(x)
-1	$-\pi/2$	π	$-\pi/4$
$-\sqrt{3}/2$	$-\pi/3$	$\frac{5\pi}{6}$	
$-1/2$	$-\pi/6$	$2\pi/3$	
$-\sqrt{2}/2$	$-\pi/4$	$3\pi/4$	
1	$\pi/2$	0	$\pi/4$
$-\sqrt{3}$			$-\pi/3$

$y \cdot \frac{\sqrt{3}}{2} \cdot \frac{2}{1} x$



6.3 **2.** Solve the equations algebraically.

a. $\sec(\theta) = 2$. Find all solutions.

$\frac{1}{\cos\theta} = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3} + 2\pi k, \quad \theta = \frac{4\pi}{3} + 2\pi k$

b. $6\sin^{-1}(x) = \pi$. Find all solutions.

$\sin^{-1}(x) = \frac{\pi}{6}$

$x = \frac{1}{2}$

c. $2\sin^2(t) + 1 = 3$ over $-2\pi \le t \le 2\pi$.

$2\sin^2 t = 2$

$\sin^2 t = 1$

$\sin t = \pm 1$

$t = \sin^{-1}(\pm 1)$

$\left\{\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}\right\}$

d. $2\cos\left(3\theta + \frac{\pi}{6}\right) = \sqrt{3}$ over $-\pi \le \theta \le 0$.

$\cos\left(3\theta + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$3\theta + \frac{\pi}{6} = \frac{\pi}{6}$

$\theta = 0$

$3\theta + \frac{\pi}{6} = -\frac{\pi}{6}$

$3\theta = -\frac{\pi}{3}$

$\theta = -\frac{\pi}{9}$

$3\theta + \frac{\pi}{6} = -\frac{11\pi}{6}$

$3\theta = -2\pi$

$\theta = -\frac{2\pi}{3}$

$3\theta + \frac{\pi}{6} = -\frac{13\pi}{6}$

$3\theta = -\frac{7\pi}{3}$

$\theta = -\frac{7\pi}{9}$

$\left\{0, -\frac{\pi}{9}, -\frac{2\pi}{3}, -\frac{7\pi}{9}\right\}$

e. $\sin(t) + 1 = 2\cos^2(t)$ over $0 \le t \le \pi$.

$\sin + 1 = 2(1 - \sin^2)$

$\sin + 1 = 2 - 2\sin^2$

$2\sin^2 + \sin - 1 = 0$

$(2\sin - 1)(\sin + 1) = 0$

$2\sin - 1 = 0 \quad \sin + 1 = 0$

$\sin = \frac{1}{2} \quad \sin = -1$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

f. $\sin(t) = \frac{1}{2}$. Find all solutions.

$t = \frac{\pi}{6} + 2\pi k, \quad \frac{5\pi}{6} + 2\pi k$

g. $\cos(2\theta) = \cos(\theta)$ over $0 \le t \le 2\pi$.

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos + 1)(\cos - 1) = 0$

$2\cos + 1 = 0 \quad \cos - 1 = 0$

$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi\right\}$

h. $3 - \sin(\theta) = \cos(2\theta)$ over $0 \le t \le 2\pi$.

$3 - \sin = 1 - 2\sin^2$

$2\sin^2 - \sin + 2 = 0$

$(2\sin \quad)(\sin \quad)$

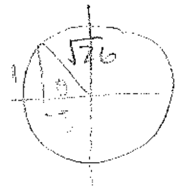
No solution

6.2 3. Evaluate the expression algebraically.

a. $\tan\left(\cos^{-1}\left(-\frac{1}{5}\right)\right)$

$= -2\sqrt{6}$

b. $\cos\left(\tan^{-1}\left(-\frac{1}{5}\right)\right)$



$= \frac{-5\sqrt{26}}{26}$

6.4 4. Prove the identities.

a. $\frac{(\csc+1)}{(\csc+1)} \cdot \frac{\csc(x)-1}{\cot(x)} = \frac{\cot(x)}{\csc(x)+1}$

$\frac{\csc^2 - 1}{\cot(\csc+1)} =$

$\frac{\cot^2}{\cot(\csc+1)} =$

$\frac{\cot}{\csc+1} = \frac{\cot}{\csc+1}$

b. $\frac{(1-\sin)}{(1-\sin)} \cdot \frac{1-\sin(x)}{\cos(x)} + \frac{\cos(x)}{1-\sin(x)} \cdot \frac{(\cos)}{(\cos)} = 2\sec(x)$

$\frac{1-2\sin x+\sin^2 + \cos^2}{\cos(1-\sin)} =$

$\frac{2-2\sin}{\cos(1-\sin)} =$

$\frac{2(1-\sin)}{\cos(1-\sin)} =$

$\frac{2}{\cos} =$

$2\sec = 2\sec$

c. $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin(\alpha)\cos(\beta)$

$\sin\alpha\cos\beta + \cos\alpha\sin\alpha + \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$2\sin\alpha\cos\beta = 2\sin\alpha\cos\beta$

d. $\frac{\cot(\theta)-\tan(\theta)}{\cot(\theta)+\tan(\theta)} = \cos(2\theta)$

$\frac{\frac{\cos}{\sin} - \frac{\sin}{\cos}}{\frac{\cos}{\sin} + \frac{\sin}{\cos}} =$

$\frac{\sin\cos}{\cos^2+\sin^2} \cdot \frac{\cos^2-\sin^2}{\sin\cos} =$

$\frac{\cos^2-\sin^2}{\cos^2+\sin^2}$

$\frac{\cos(2\theta)}{1} =$

$\cos(2\theta) = \cos(2\theta)$

$\cos(\alpha) = \frac{-\sqrt{21}}{5}$ $\cos(\beta) = \frac{-1}{3}$ $\sin\beta = \frac{2\sqrt{2}}{3}$

6.5 6.6 **5.** If $\sin(\alpha) = -\frac{2}{5}$ where $\pi \le \alpha \le \frac{3\pi}{2}$, and $\sec(\beta) = -3$ where $\frac{\pi}{2} \le \beta \le \pi$, then find

a. $\sin(2\alpha)$ **b.** $\cos(\alpha+\beta)$ **c.** $\cos(2\beta)$ **d.** $\sin(\alpha-\beta)$ **e.** $\cot(\beta)$. $= \frac{\cos\beta}{\sin\beta}$

a. $= 2\sin\alpha\cos\alpha$

$= 2 \cdot -\frac{2}{5} \cdot \frac{-\sqrt{21}}{5}$

$= \frac{4\sqrt{21}}{25}$

b. $= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{-\sqrt{21}}{5} \cdot \frac{-1}{3} - \frac{-2}{5} \cdot \frac{2\sqrt{2}}{3}$

$= \frac{\sqrt{21}}{15} + \frac{4\sqrt{2}}{15}$

$= \frac{\sqrt{21}+4\sqrt{2}}{15}$

c. $= 2\cos^2\beta - 1$

$= 2\left(\frac{-1}{3}\right)^2 - 1$

$= 2 \cdot \frac{1}{9} - 1$

$= \frac{2}{9} - \frac{9}{9}$

$= -\frac{7}{9}$

d. $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= -\frac{2}{5} \cdot \frac{-1}{3} - \frac{-\sqrt{21}}{5} \cdot \frac{2\sqrt{2}}{3}$

$= \frac{2}{15} + \frac{2\sqrt{42}}{15}$

$= \frac{2+2\sqrt{42}}{15}$

e. $= \frac{-1}{3} \cdot \frac{3}{2\sqrt{2}}$

$= \frac{-1}{2\sqrt{2}}$

$= \frac{-\sqrt{2}}{4}$

6.5 **6.** Find the exact value of

a. sin(75)

$= \sin(45+30)$

$= \sin 45\cos 30 + \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

b. $\cos\left(\frac{11\pi}{12}\right)$

$= \cos\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{-\sqrt{2}-\sqrt{6}}{4}$

6.7 **7.** Write the product $\sin(165°)\sin(75°)$ as a sum and evaluate.

$= \frac{1}{2}\left[\cos(90) - \cos(240)\right]$

$= \frac{1}{2}\left(0 + \frac{1}{2}\right)$

$= \frac{1}{4}$

8. Solve $\sin^2(\theta) = \frac{1}{9}$ over $0 \le t \le 2\pi$. Round to the nearest thousandth. OOPS! CALC. REQUIRED

$\sin\theta = \pm\frac{1}{\sqrt{3}}$

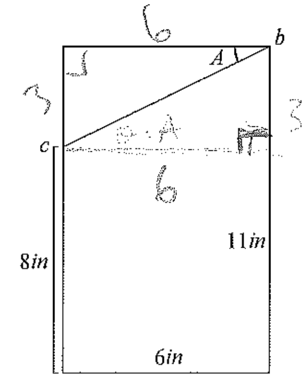
$\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\theta = 0.615$

Math 112 Name:______________________

MTH 112 Exam 1 Review
Calculator Portion

1. A birdhouse is shown in profile, without the roof. The desired dimensions are shown. To what angle, A, do I set my saw so that it cuts from b to c.

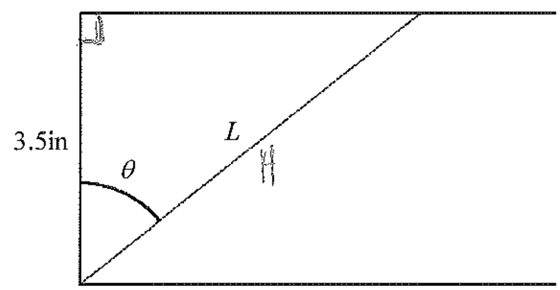


$\tan\theta = \frac{3}{6}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta = 26.6°$

2. If you take your average 2x4x8 lumber (actual dimensions 1.5in thick by 3.5in wide by 8ft long) and cut it at an angle θ, as pictured, what will the length of the cut face, L, be?



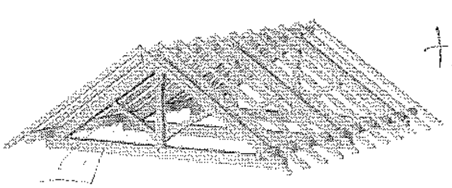
$\cos\theta = \frac{3.5}{L}$ $\quad L = \frac{3.5}{\cos\theta}$

a. If you cut off an angle of 36°, what will be the length of L?

$L = \frac{3.5}{\cos 36°}$

$L = 4.33$ inches

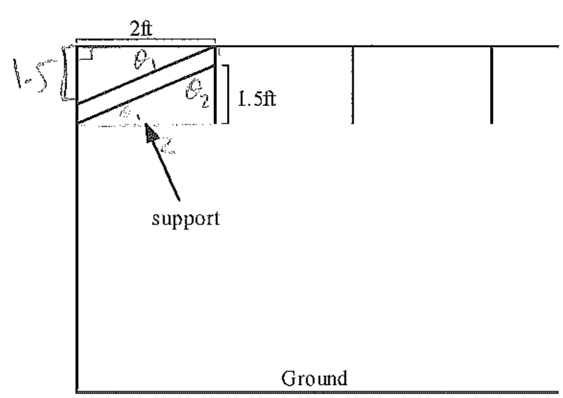
3. In my greenhouse, I have a trussed roof (with photo below as an example) that needs support front and back. Shown right is a side view with support added. Solve for all the angles in the picture, assuming that all of the boards were attached at right angles besides the support board shown.



$\tan(\theta_1) = \frac{1.5}{2}$

$\theta_1 = \tan^{-1}\left(\frac{1.5}{2}\right)$

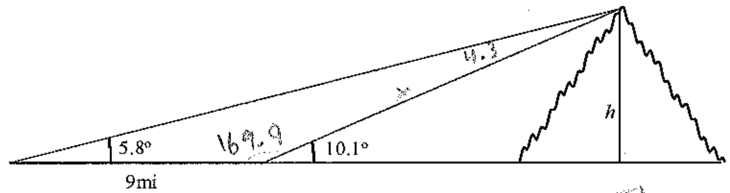
$\theta_1 \approx 36.9°$



$\theta_2 = 90 - 36.9$

$\theta_2 \approx 53.1°$

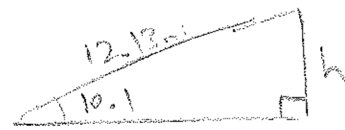
7.2 **4.** After a rafting trip on the Descutes River you drive back to Portland. Mount Hood stands in your way. Since you're in trig class, you have your handy protractor with you and you measure that where you are on the road, the tallest peak of Mt Hood is at an elevation of about $5.8°$. You hop back in your car and drive straight toward Mt Hood again for 9mi. Checking here, Mt Hood's peak is now at an angle of $10.1°$. From this, you use your cell phone's calculator to figure out Mt Hood's height, h. When you get back into cell range, you check your answer. Redo this calculation so you can prove that you're that good at trig. Shown is a picture of the scenario, not to scale.



$$\frac{\sin 4.3}{9} = \frac{\sin 5.8}{x}$$

$$x \cdot \sin(4.3) = 9 \cdot \sin(5.8)$$

$$x = 12.13 \text{ mi}$$

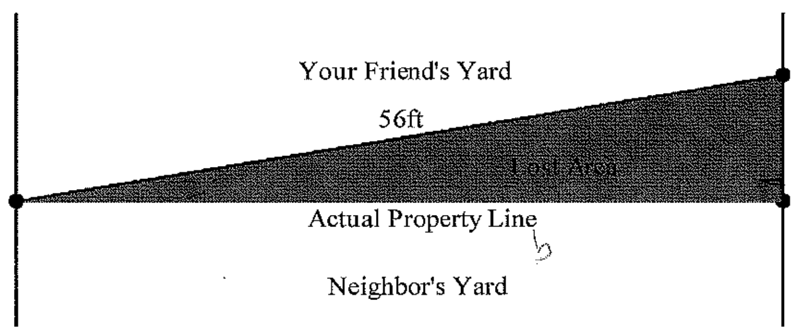


$$\sin(10.1) = \frac{h}{12.13}$$

$$h = 12.13 \cdot \sin(10.1)$$

$$h \approx 2.13 \text{ miles}$$

7.2 **5.** Imagine building a fence along the property border in a friend's yard. They like to cut corners and not measure properly. Their fence, as he built it, was 56ft long, and matched up with another fence at the side of his property; however, their measurements were off by $2°$, from a birds-eye-view, angled into his own property, as shown. What area of land did he lose to his neighbor due to his careless measurements? Round your answer to the nearest square foot.



$$\sin 2 = \frac{h}{56} \qquad \cos 2 = \frac{b}{56}$$

$$h = 56 \sin 2 \qquad b = 56 \cos 2$$

$$h = 1.9544 \text{ ft} \qquad b = 55.9659 \text{ ft}$$

$$A = \frac{1.9544 \cdot 55.9659}{2}$$

$$A \approx 54.689 \text{ sq. ft}$$