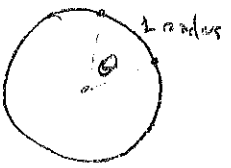
1. a. $f(1) = 0$ b. No, it's not 1-1 c. $f(x) = 1$ $\{\frac{1}{2}, 3\}$

d. $x \leq 0$ e. $(-5,-3) \cup (1,3)$ f. D: $[-5,5]$ g. R: $[-2,5]$

h. $(-3,5)$

2. 1 radian is the angle swept out along a circle when the radius is placed along the circumference.



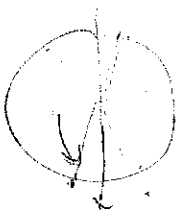
3. a. F b. F c. F d. F e. T f. F g. F

h. F i. T j. F k. T l. F

4. a. $75° \times \left(\frac{\pi}{180°}\right) = 1.31$ radians

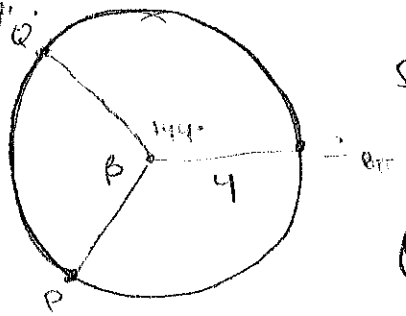
b. $\frac{4\pi}{9} \times \left(\frac{180°}{\pi}\right) = 80°$

Math 112
FINAL REVIEW

5.  60°

6.  160° and 380°

7.



$s = \frac{28\pi}{5} \times \frac{1}{8\pi} = 0.7$ of the circle

$\theta = 0.7 \times 360° = 252°$

$\beta = 252 - 144 = 108°$

$\beta = 108° \left(\frac{\pi}{180°}\right) = 1.8850$ radians

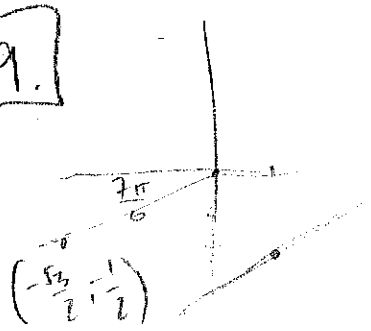
8. $\csc\left(\frac{4\pi}{3}\right) = \frac{-2}{\sqrt{3}}$ $\sec\left(\frac{5\pi}{6}\right) = \frac{-2}{\sqrt{3}}$ $\cot\left(\frac{7\pi}{4}\right) = 1$

$$\frac{\csc\left(\frac{4\pi}{3}\right) + \sec\left(\frac{5\pi}{6}\right)}{\cot\left(\frac{7\pi}{4}\right)} = \frac{\frac{-2}{\sqrt{3}} - \frac{2}{\sqrt{3}}}{1}$$

$$= \frac{-4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{-4\sqrt{3}}{3}$$

9.

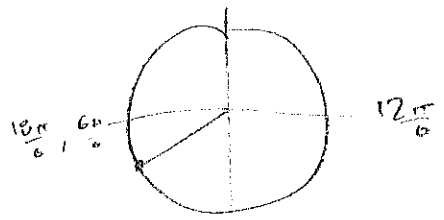


$m = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

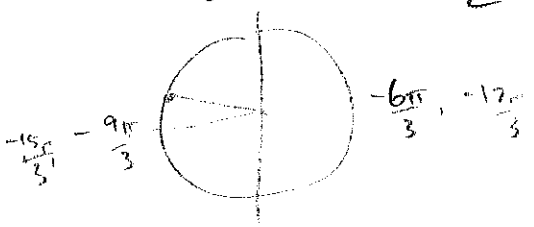
$m = \frac{\sqrt{3}}{3}$

$y + 2 = \frac{\sqrt{3}}{3}(x - 1)$

10. a. $\cos\left(\frac{19\pi}{6}\right) = -\frac{\sqrt{3}}{2}$



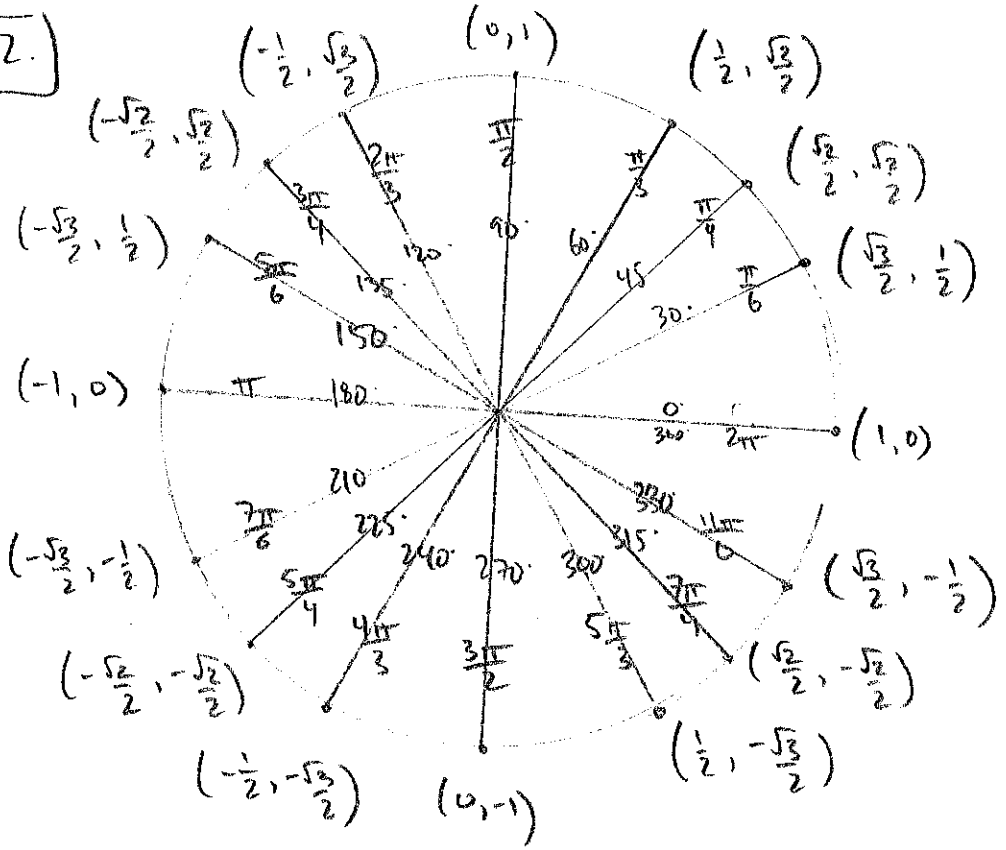
b. $\cos\left(\frac{-16\pi}{3}\right) = -\frac{\sqrt{3}}{2}$



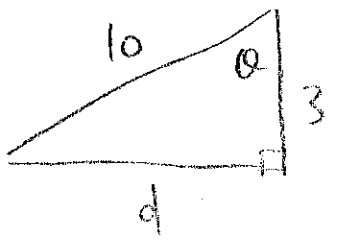
11. $y = \tan(\theta)$

D: $\{\theta \mid \theta \neq \frac{\pi}{2} + 2\pi k\}$

12.



13.



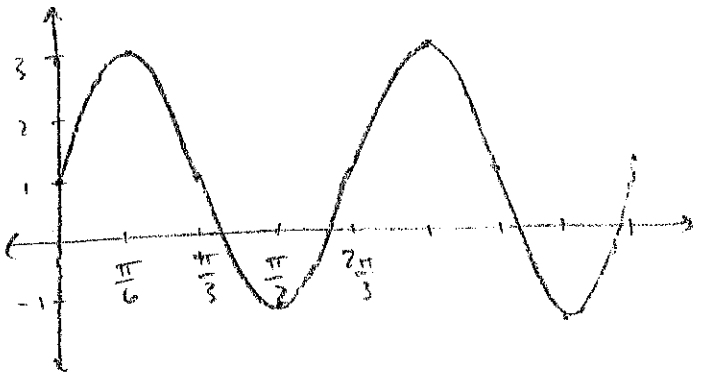
$\cos\theta = \frac{3}{10}$ $\sin\theta = \frac{\sqrt{91}}{10}$

$d = \sqrt{91}$

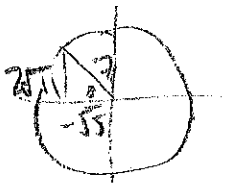
14. Amplitude: 2

midline: $y = 1$

Period: $\frac{2\pi}{3}$



15. a. $\cos\theta = -\frac{\sqrt{5}}{7}$



$y = \sqrt{7^2 - (-\sqrt{5})^2}$

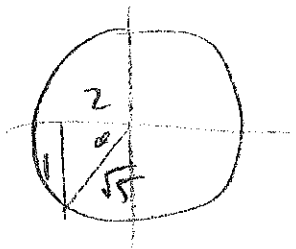
$y = \sqrt{49 - 5}$

$y = \sqrt{44} = 2\sqrt{11}$

$\sin\theta = \frac{2\sqrt{11}}{7}$

$\tan\theta = \frac{-2\sqrt{55}}{5}$

b. $\cot\theta = 2$



$\sin\theta = \frac{-1}{\sqrt{5}}$

$= -\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{1}{2}$

16. $y = -6\sin\left[\frac{2\pi}{3}(x-1)\right]+1$ $\quad y = 6\cos\left[\frac{2\pi}{3}\left(x-\frac{1}{3}\right)\right]+1$

$\frac{2\pi}{\omega} = 3$

17.

x	$\sin^{-1}(x)$	$\cos^{-1}(x)$	$\tan^{-1}(x)$
-1	$-\frac{\pi}{2}$	π	$-\frac{\pi}{4}$
$-\frac{\sqrt{3}}{2}$	$-\frac{\pi}{3}$	$\frac{5\pi}{6}$	
$-\frac{1}{2}$	$-\frac{\pi}{6}$	$2\pi/3$	
$-\frac{\sqrt{2}}{2}$	$-\frac{\pi}{4}$	$\frac{3\pi}{4}$	
1	$\frac{\pi}{2}$	0	$\frac{\pi}{4}$
$-\sqrt{3}$			$-\frac{\pi}{3}$

18. a. QII b. QIV c. QI

19. a. $\sec(\theta) = 2$

$\cos(\theta) = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\left\{\frac{\pi}{3}+2\pi k, \frac{5\pi}{3}+2\pi k\right\}$

b. $\sin(2\theta) = \cos(\theta)$

$\frac{2\sin\theta\cos\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

c. $6\sin^{-1}(x) = \pi$

$\sin^{-1}(x) = \frac{\pi}{6}$

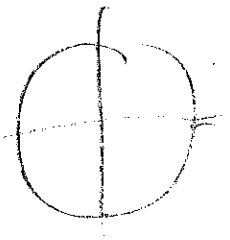
$x = \frac{1}{2}$

d. $2\sin^2(t) + 1 = 3$

$2\sin^2(t) = 2$

$\sin^2(t) = 1$

$\sin(t) = \pm 1$



$\left\{-\frac{\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

e. $\sec^2\theta + \tan\theta = 0$

$\tan^2\theta + 1 + \tan\theta = 0$

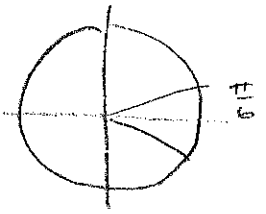
$\tan^2\theta + \tan\theta + 1 = 0$

No solution

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

f. $2\cos\left(3\theta + \frac{\pi}{6}\right) = \sqrt{3}$ over $-\pi \le \theta \le 0$



$\cos\left(3\theta + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$3\theta + \frac{\pi}{6} = \frac{\pi}{6} + 2\pi k$ or $-\frac{\pi}{6} + 2\pi k$

$3\theta + \frac{\pi}{6} = \frac{\pi}{6}$

$3\theta = 0$

$\theta = 0$

$3\theta + \frac{\pi}{6} = -\frac{\pi}{6}$

$3\theta = -\frac{2\pi}{3} \cdot \frac{1}{3}$

$\theta = -\frac{\pi}{9}$

$3\theta + \frac{\pi}{6} = -\frac{11\pi}{6}$

$3\theta = -\frac{12\pi}{6} \cdot \frac{1}{3}$

$\theta = -\frac{2\pi}{3}$

$3\theta + \frac{\pi}{6} = -\frac{13\pi}{6}$

$\frac{1}{3}3\theta = -\frac{14\pi}{6} \cdot \frac{1}{3} = -\frac{14\pi}{18}$

$\theta = -\frac{7\pi}{9}$

$\left\{-\frac{7\pi}{9}, -\frac{2\pi}{3}, -\frac{\pi}{9}, 0\right\}$

g. $\sin(t)+1=2\cos^2(t)$

$\sin(t)+1=2(1-\sin^2(t))$

$\sin(t)+1=2-2\sin^2(t)$

$2\sin^2(t)+\sin(t)-1=0$

$(2\sin(t)-1)(\sin(t)+1)=0$

$2\sin(t)-1=0$ $\quad$ $\sin(t)+1=0$

$\sin(t)=\frac{1}{2}$ $\quad$ $\sin(t)=-1$

$\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

h. $\sin\left(\frac{t}{3}\right)=\frac{1}{2}$ for $0\le t\le 2\pi$

$\frac{t}{3}=\frac{\pi}{6}$ $\quad$ $\frac{t}{3}=\frac{5\pi}{6}$

$t=\frac{\pi}{2}$ $\quad$ $t=\frac{15\pi}{6}$

$\left\{\frac{\pi}{2}\right\}$

i $\sin(t)=\frac{1}{2}$

$\left\{\frac{\pi}{6}+2\pi k, \frac{5\pi}{6}+2\pi k\right\}$

j. $\cos(2\theta)=\cos\theta$

$2\cos^2\theta-1=\cos\theta$

$2\cos^2\theta-\cos\theta-1=0$

$(2\cos\theta+1)(\cos\theta-1)=0$

$2\cos\theta+1=0$ $\quad$ $\cos\theta-1=0$

$\cos\theta=-\frac{1}{2}$ $\quad$ $\cos\theta=1$

$\left\{\frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi\right\}$

k. $3-\sin(\theta)=\cos(2\theta)$ for $0\le\theta\le 2\pi$

$3-\sin\theta=1-2\sin^2\theta$

$2\sin^2\theta-\sin\theta+2=0$

No solution

19. $\sin(4\theta) - \sin(2\theta) = 0$

$2\cos\left(\frac{4\theta+2\theta}{2}\right)\sin\left(\frac{4\theta-2\theta}{2}\right) = 0$

$2\cdot\cos(3\theta)\sin(\theta) = 0$

$\cos 3\theta = 0 \qquad \sin\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \qquad \theta = 0, 2\pi$

$3\theta = \frac{\pi}{2} \qquad 3\theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{6} \qquad \theta = \frac{\pi}{2}$

$3\theta = \frac{5\pi}{2} \qquad 3\theta = \frac{7\pi}{2}$

$\theta = \frac{5\pi}{6} \qquad \theta = \frac{7\pi}{6}$

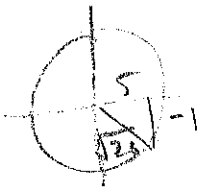
$3\theta = \frac{9\pi}{2} \qquad 3\theta = \frac{11\pi}{2}$

$\theta = \frac{9\pi}{6} = \frac{3\pi}{2} \qquad \theta = \frac{11\pi}{6}$

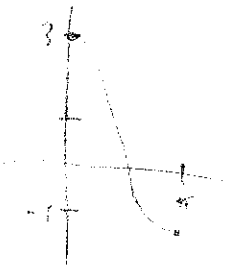
$\left\{0, 2\pi, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

20. a. $\tan\left(\cos^{-1}\left(-\frac{1}{5}\right)\right) = -\sqrt{24}$

b. $\cos\left(\tan^{-1}\left(-\frac{1}{5}\right)\right) = \frac{-\sqrt{26}}{26}$



21. $f(x) = 2\cos(x) + 1$



$x = 2\cos(y) + 1$

$\frac{x-1}{2} = \cos(y)$

$f^{-1}(x) = \cos^{-1}\left(\frac{x-1}{2}\right)$ over $-1 \le x \le 3$

b.

$f(x)$	$f^{-1}(x)$
$D: [0, \pi]$	$D: [-1, 3]$
$R: [-1, 3]$	$R: [0, \pi]$

22. a. $\frac{(\csc(x)+1)\csc(x)-1}{(\csc(x)+1)\cot(x)} = \frac{\cot(x)}{\csc(x)+1}$

$\frac{\csc^2(x)-1}{\cot(\csc(x)+1)} =$

$\frac{\cot^2(x)}{\cot(x)(\csc(x)+1)} =$

$\frac{\cot(x)}{\csc(x)+1} = \frac{\cot(x)}{\csc(x)+1}$

b. $\frac{1-\sin(x)}{\cos(x)} + \frac{\cos(x)}{1-\sin(x)} = 2\sec(x)$

$\frac{(1-\sin(x))^2 + \cos^2(x)}{\cos(x)(1-\sin(x))} =$

$\frac{1-2\sin(x)+\sin^2(x)+\cos^2(x)}{\cos(x)(1-\sin(x))} =$

$\frac{2-2\sin(x)}{\cos(x)(1-\sin(x))} =$

$\frac{2(1-\sin(x))}{\cos(x)(1-\sin(x))}$

$\frac{2}{\cos(x)} =$

$2\sec(x) = 2\sec(x)$

c. $\ln|\tan(w)| = \ln\left|\frac{\sin w}{\cos w}\right|$

$\ln|\tan(w)| = \ln|\tan(w)|$

d. $\sin(\alpha+\beta)+\sin(\alpha-\beta) = 2\sin(\alpha)\cos(\beta)$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$2\sin(\alpha)\cos(\beta) = 2\sin(\alpha)\cos(\beta)$

e. $\frac{\cot(\theta)-\tan(\theta)}{\cot(\theta)+\tan(\theta)} = \cos(2\theta)$

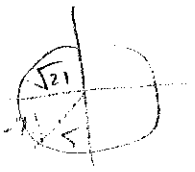
$\frac{\frac{\cos}{\sin} - \frac{\sin}{\cos}}{\frac{\cos}{\sin} + \frac{\sin}{\cos}} =$

$\frac{\cos^2-\sin^2}{\sin\cos} \cdot \frac{\sin\cos}{\cos^2+\sin^2} =$

$\cos^2-\sin^2 =$

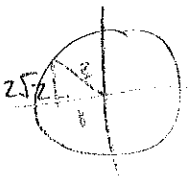
$\cos(2\theta) = \cos(2\theta)$

23. $\sin\alpha = \frac{-2}{5}$



$\cos\alpha = -\frac{\sqrt{21}}{5}$

$\cos\beta = -\frac{1}{3}$



$\sin\beta = \frac{2\sqrt{2}}{3}$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$= 2\left(-\frac{2}{5}\right)\left(-\frac{\sqrt{21}}{5}\right)$

$= \frac{4\sqrt{21}}{25}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= -\frac{\sqrt{21}}{5}\left(-\frac{1}{3}\right) - \frac{-2}{5}\left(\frac{2\sqrt{2}}{3}\right)$

$= \frac{\sqrt{21}}{15} + \frac{4\sqrt{2}}{15}$

$= \frac{1}{15}\left(\sqrt{21} + 4\sqrt{2}\right)$

$\cos(2\beta) = 2\cos^2\beta - 1$

$= 2\left(-\frac{1}{3}\right)^2 - 1$

$= \frac{2}{9} - 1$

$= -\frac{7}{9}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{-2}{5}\left(-\frac{1}{3}\right) - \frac{-\sqrt{21}}{5}\left(\frac{2\sqrt{2}}{3}\right)$

$= \frac{2}{15} + \frac{2\sqrt{42}}{15}$

$= \frac{1}{15}\left(2 + 2\sqrt{42}\right)$

$\cot(\beta) = \frac{\cos\beta}{\sin\beta}$

$= -\frac{1}{3} \cdot \frac{3}{2\sqrt{2}}$

$= \frac{-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{-\sqrt{2}}{4}$

24. $\sin(75^\circ) = \sin(30+45)$

$= \sin 30\cos 45 + \cos 30 \sin 45$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

b. $\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{9\pi}{12} + \frac{2\pi}{12}\right)$

$= \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

25. $\sin(165^\circ)\sin(75^\circ) = \frac{1}{2}\left[\cos(90) - \cos(240)\right]$

$= \frac{1}{2}\left[0 + \frac{1}{2}\right]$

$= \frac{1}{4}$

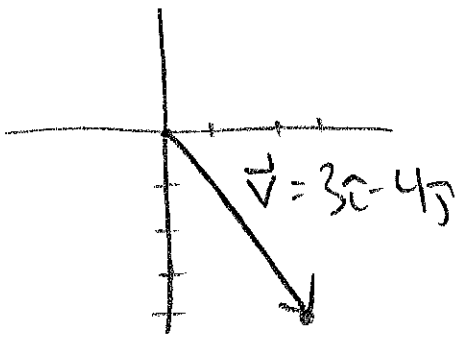
26. $\vec{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ $\quad \vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$

a. $2\vec{a} - \vec{b} = 6\hat{i} + 8\hat{j} - 4\hat{k} + \hat{i} - 2\hat{j} - 3\hat{k}$

$= 7\hat{i} + 6\hat{j} - 7\hat{k}$

b. Don't do.

29.



$\|\vec{v}\| = \sqrt{3^2 + 4^2}$

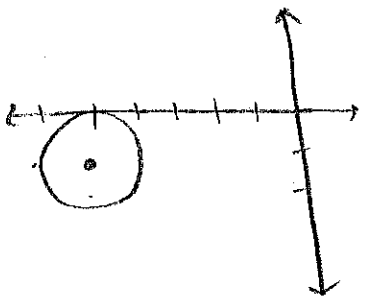
$\|\vec{v}\| = 5$

31. $(x+2)^2 + (y-3)^2 = 16$

32. $x^2+10x+y^2+4y = -28$

$x^2+10x+\underline{25} + y^2+4y+\underline{4} = -28+25+4$

$(x+5)^2 + (y+2)^2 = 1$



center: $(-5,-2)$

$r = 1$

34. $\vec{v} = 2\hat{i} + \frac{1}{2}\hat{j}$ $\vec{w} = 3\hat{j}$ $\vec{u} = \hat{i}+4\hat{j}$

$-2\vec{w} = -6\hat{j}$ $3\vec{u} = 3\hat{i}+12\hat{j}$

$\vec{v} - 2\vec{w} + 3\vec{u} = 2\hat{i} + \frac{1}{2}\hat{j} - 6\hat{j} + 3\hat{i} + 12\hat{j}$

$= 5\hat{i} + 6\frac{1}{2}\hat{j}$