4.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Find $f(3)$ if $f(x) = -4x^2 + 5x$. (pp. 46–49)
2. Find $f(3x)$ if $f(x) = 4 - 2x^2$. (pp. 46–49)
3. Find the domain of the function $f(x) = \dfrac{x^2 - 1}{x^2 - 25}$. (pp. 49–51)

Concepts and Vocabulary

4. Given two functions f and g, the ________ ________, denoted $f \circ g$, is defined by $(f \circ g)(x) =$ ________.
5. ***True or False*** $f(g(x)) = f(x) \cdot g(x)$.
6. ***True or False*** The domain of the composite function $(f \circ g)(x)$ is the same as the domain of $g(x)$.

Skill Building

In Problems 7 and 8, evaluate each expression using the values given in the table.

7.

x	−3	−2	−1	0	1	2	3
$f(x)$	−7	−5	−3	−1	3	5	7
$g(x)$	8	3	0	−1	0	3	8

(a) $(f \circ g)(1)$ (b) $(f \circ g)(-1)$
(c) $(g \circ f)(-1)$ (d) $(g \circ f)(0)$
(e) $(g \circ g)(-2)$ (f) $(f \circ f)(-1)$

8.

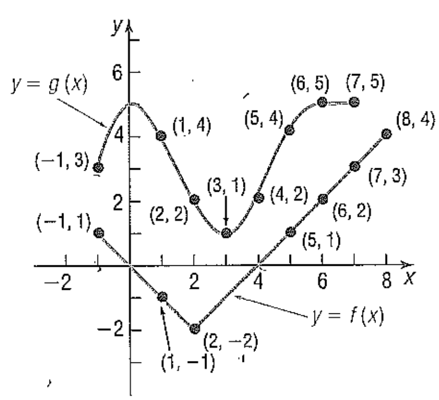
x	−3	−2	−1	0	1	2	3
$f(x)$	11	9	7	5	3	1	−1
$g(x)$	−8	−3	0	1	0	−3	−8

(a) $(f \circ g)(1)$ (b) $(f \circ g)(2)$
(c) $(g \circ f)(2)$ (d) $(g \circ f)(3)$
(e) $(g \circ g)(1)$ (f) $(f \circ f)(3)$

In Problems 9 and 10, evaluate each expression using the graphs of $y = f(x)$ and $y = g(x)$ shown in the figure.

9. (a) $(g \circ f)(-1)$ (b) $(g \circ f)(0)$
(c) $(f \circ g)(-1)$ (d) $(f \circ g)(4)$

10. (a) $(g \circ f)(1)$ (b) $(g \circ f)(5)$
(c) $(f \circ g)(0)$ (d) $(f \circ g)(2)$



In Problems 11–20, for the given functions f and g, find:
(a) $(f \circ g)(4)$ (b) $(g \circ f)(2)$ (c) $(f \circ f)(1)$ (d) $(g \circ g)(0)$

11. $f(x) = 2x$; $g(x) = 3x^2 + 1$
12. $f(x) = 3x + 2$; $g(x) = 2x^2 - 1$
13. $f(x) = 4x^2 - 3$; $g(x) = 3 - \frac{1}{2}x^2$
14. $f(x) = 2x^2$; $g(x) = 1 - 3x^2$
15. $f(x) = \sqrt{x}$; $g(x) = 2x$
16. $f(x) = \sqrt{x + 1}$; $g(x) = 3x$
17. $f(x) = |x|$; $g(x) = \dfrac{1}{x^2 + 1}$
18. $f(x) = |x - 2|$; $g(x) = \dfrac{3}{x^2 + 2}$
19. $f(x) = \dfrac{3}{x + 1}$; $g(x) = \sqrt[3]{x}$
20. $f(x) = x^{3/2}$; $g(x) = \dfrac{2}{x + 1}$

In Problems 21–30, find the domain of the composite function $f \circ g$.

21. $f(x) = \dfrac{3}{x - 1}$; $g(x) = \dfrac{2}{x}$
22. $f(x) = \dfrac{1}{x + 3}$; $g(x) = -\dfrac{2}{x}$

23. $f(x) = \frac{x}{x-1}$; $g(x) = -\frac{4}{x}$

24. $f(x) = \frac{x}{x+3}$; $g(x) = \frac{2}{x}$

25. $f(x) = \sqrt{x}$; $g(x) = 2x + 3$

26. $f(x) = x - 2$; $g(x) = \sqrt{1-x}$

27. $f(x) = x^2 + 1$; $g(x) = \sqrt{x-1}$

28. $f(x) = x^2 + 4$; $g(x) = \sqrt{x-2}$

29. $f(x) = \frac{x-6}{x-2}$; $g(x) = \sqrt{x}$

30. $f(x) = \sqrt{x}$; $g(x) = \frac{x}{x-3}$

In Problems 31–46, for the given functions f and g, find:
(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$
State the domain of each composite function.

31. $f(x) = 2x + 3$; $g(x) = 3x$

32. $f(x) = -x$; $g(x) = 2x - 4$

33. $f(x) = 3x + 1$; $g(x) = x^2$

34. $f(x) = x + 1$; $g(x) = x^2 + 4$

35. $f(x) = x^2$; $g(x) = x^2 + 4$

36. $f(x) = x^2 + 1$; $g(x) = 2x^2 + 3$

37. $f(x) = \frac{3}{x-1}$; $g(x) = \frac{2}{x}$

38. $f(x) = \frac{1}{x+3}$; $g(x) = -\frac{2}{x}$

39. $f(x) = \frac{x}{x-1}$; $g(x) = -\frac{4}{x}$

40. $f(x) = \frac{x}{x+3}$; $g(x) = \frac{2}{x}$

41. $f(x) = \sqrt{x}$; $g(x) = 2x + 3$

42. $f(x) = \sqrt{x-2}$; $g(x) = 1 - 2x$

43. $f(x) = x^2 + 1$; $g(x) = \sqrt{x-1}$

44. $f(x) = x^2 + 4$; $g(x) = \sqrt{x-2}$

45. $f(x) = \frac{x-5}{x+1}$; $g(x) = \frac{x+2}{x-3}$

46. $f(x) = \frac{2x-1}{x-2}$; $g(x) = \frac{x+4}{2x-5}$

In Problems 47–54, show that $(f \circ g)(x) = (g \circ f)(x) = x$.

47. $f(x) = 2x$; $g(x) = \frac{1}{2}x$

48. $f(x) = 4x$; $g(x) = \frac{1}{4}x$

49. $f(x) = x^3$; $g(x) = \sqrt[3]{x}$

50. $f(x) = x + 5$; $g(x) = x - 5$

51. $f(x) = 2x - 6$; $g(x) = \frac{1}{2}(x + 6)$

52. $f(x) = 4 - 3x$; $g(x) = \frac{1}{3}(4 - x)$

53. $f(x) = ax + b$; $g(x) = \frac{1}{a}(x - b)$ $a \neq 0$

54. $f(x) = \frac{1}{x}$; $g(x) = \frac{1}{x}$

In Problems 55–60, find functions f and g so that $f \circ g = H$.

55. $H(x) = (2x + 3)^4$

56. $H(x) = (1 + x^2)^3$

57. $H(x) = \sqrt{x^2 + 1}$

58. $H(x) = \sqrt{1 - x^2}$

59. $H(x) = |2x + 1|$

60. $H(x) = |2x^2 + 3|$

Applications and Extensions

61. If $f(x) = 2x^3 - 3x^2 + 4x - 1$ and $g(x) = 2$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

62. If $f(x) = \frac{x+1}{x-1}$, find $(f \circ f)(x)$.

63. If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 23.

64. If $f(x) = 3x^2 - 7$ and $g(x) = 2x + a$, find a so that the graph of $f \circ g$ crosses the y-axis at 68.

In Problems 65 and 66, use the functions f and g to find:
(a) $f \circ g$
(b) $g \circ f$
(c) the domain of $f \circ g$ and of $g \circ f$
(d) the conditions for which $f \circ g = g \circ f$

65. $f(x) = ax + b$; $g(x) = cx + d$

66. $f(x) = \frac{ax+b}{cx+d}$; $g(x) = mx$

67. **Surface Area of a Balloon** The surface area S (in square meters) of a hot-air balloon is given by

$$S(r) = 4\pi r^2$$

where r is the radius of the balloon (in meters). If the radius r is increasing with time t (in seconds) according to the formula $r(t) = \frac{2}{3}t^3, t \geq 0$, find the surface area S of the balloon as a function of the time t.

68. **Volume of a Balloon** The volume V (in cubic meters) of the hot-air balloon described in Problem 67 is given by $V(r) = \frac{4}{3}\pi r^3$. If the radius r is the same function of t as in Problem 67, find the volume V as a function of the time t.

69. **Automobile Production** The number N of cars produced at a certain factory in one day after t hours of operation is given by $N(t) = 100t - 5t^2, 0 \leq t \leq 10$. If the cost C (in dollars) of producing N cars is $C(N) = 15{,}000 + 8000N$, find the cost C as a function of the time t of operation of the factory.

70. **Environmental Concerns** The spread of oil leaking from a tanker is in the shape of a circle. If the radius r (in feet) of the spread after t hours is $r(t) = 200\sqrt{t}$, find the area A of the oil slick as a function of the time t.

71. Production Cost The price p, in dollars, of a certain product and the quantity x sold obey the demand equation

$$p = -\frac{1}{4}x + 100 \quad 0 \le x \le 400$$

Suppose that the cost C, in dollars, of producing x units is

$$C = \frac{\sqrt{x}}{25} + 600$$

Assuming that all items produced are sold, find the cost C as a function of the price p.

[**Hint:** Solve for x in the demand equation and then form the composite.]

72. Cost of a Commodity The price p, in dollars, of a certain commodity and the quantity x sold obey the demand equation

$$p = -\frac{1}{5}x + 200 \quad 0 \le x \le 1000$$

Suppose that the cost C, in dollars, of producing x units is

$$C = \frac{\sqrt{x}}{10} + 400$$

Assuming that all items produced are sold, find the cost C as a function of the price p.

73. Volume of a Cylinder The volume V of a right circular cylinder of height h and radius r is $V = \pi r^2 h$. If the height is twice the radius, express the volume V as a function of r.

74. Volume of a Cone The volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$. If the height is twice the radius, express the volume V as a function of r.

75. Foreign Exchange Traders often buy foreign currency in the hope of making money when the currency's value changes. For example, on April 18, 2013, one U.S. dollar could purchase 0.7643 euros, and one euro could purchase 128.4594 yen. Let $f(x)$ represent the number of euros you can buy with x dollars, and let $g(x)$ represent the number of yen you can buy with x euros.

(a) Find a function that relates dollars to euros.

(b) Find a function that relates euros to yen.

(c) Use the results of parts (a) and (b) to find a function that relates dollars to yen. That is, find $(g \circ f)(x) = g(f(x))$.

(d) What is $g(f(1000))$? Interpret this result.

76. Temperature Conversion The function $C(F) = \frac{5}{9}(F - 32)$ converts a temperature in degrees Fahrenheit, F, to a temperature in degrees Celsius, C. The function $K(C) = C + 273$ converts a temperature in degrees Celsius to a temperature in kelvins, K.

(a) Find a function that converts a temperature in degrees Fahrenheit to a temperature in kelvins.

(b) Determine 80 degrees Fahrenheit in kelvins.

77. Discounts The manufacturer of a computer is offering two discounts on last year's model computer. The first discount is a \$200 rebate and the second discount is 20% off the regular price, p.

(a) Write a function f that represents the sale price if only the rebate applies.

(b) Write a function g that represents the sale price if only the 20% discount applies.

(c) Find $f \circ g$ and $g \circ f$. What does each of these functions represent? Which combination of discounts represents a better deal for the consumer? Why?

78. Taxes Suppose that you work for \$15 per hour. Write a function that represents gross salary G as a function of hours worked h. Your employer is required to withhold taxes (federal income tax, Social Security, Medicare) from your paycheck. Suppose your employer withholds 20% of your income for taxes. Write a function that represents net salary N as a function of gross salary G. Find and interpret $N \circ G$.

79. Suppose that $f(x) = x^3 + x^2 - 16x - 16$ and $g(x) = x^2 - 4$. Find the zeros of $(f \circ g)(x)$.

80. Suppose that $f(x) = 2x^3 - 3x^2 - 8x + 12$ and $g(x) = x + 5$. Find the zeros of $(f \circ g)(x)$.

81. If f and g are odd functions, show that the composite function $f \circ g$ is also odd.

82. If f is an odd function and g is an even function, show that the composite functions $f \circ g$ and $g \circ f$ are both even.

83. Let $f(x) = ax + b$ and $g(x) = bx + a$, where a and b are integers. If $f(1) = 8$ and $f(g(20)) - g(f(20)) = -14$, find the product of a and b.*

Retain Your Knowledge

Problems 84–87 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

84. Given $f(x) = 3x + 8$ and $g(x) = x - 5$, find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$. State the domain of each.

85. Find the real zeros of $f(x) = 2x - 5\sqrt{x} + 2$.

86. Use a graphing utility to graph $f(x) = -x^3 + 4x - 2$ over the interval $(-3, 3)$. Approximate any local maxima and local minima. Determine where the function is increasing and where it is decreasing.

87. Find the domain of $R(x) = \frac{x^2 + 6x + 5}{x - 3}$. Find any horizontal, vertical, or oblique asymptotes.

'Are You Prepared?' Answers

1. -21 **2.** $4 - 18x^2$ **3.** $\{x \mid x \neq -5, x \neq 5\}$

*Courtesy of the Joliet Junior College Mathematics Department