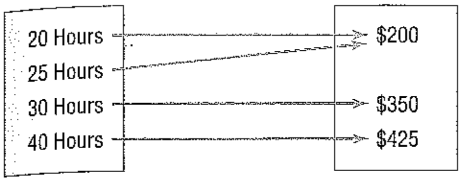
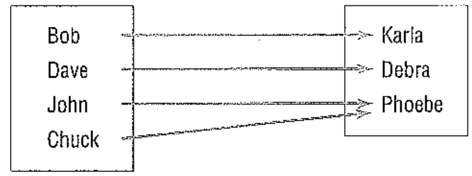
13.



14.



15. $\{(2, 6), (-3, 6), (4, 9), (1, 10)\}$

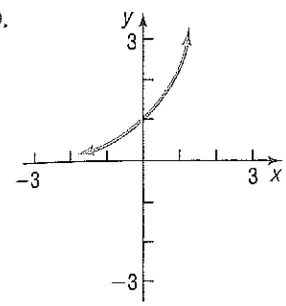
16. $\{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$

17. $\{(0, 0), (1, 1), (2, 16), (3, 81)\}$

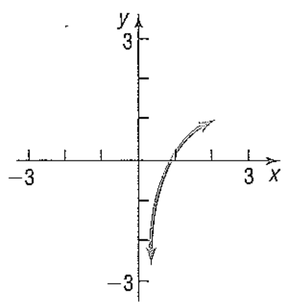
18. $\{(1, 2), (2, 8), (3, 18), (4, 32)\}$

In Problems 19–24, the graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.

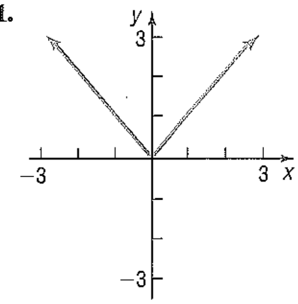
19.



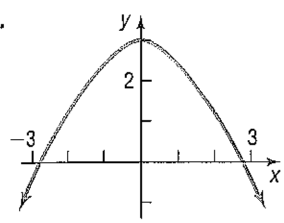
20.



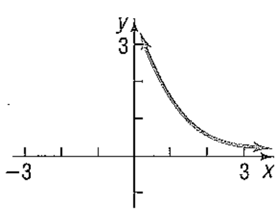
21.



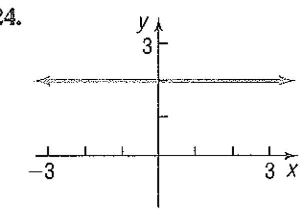
22.



23.

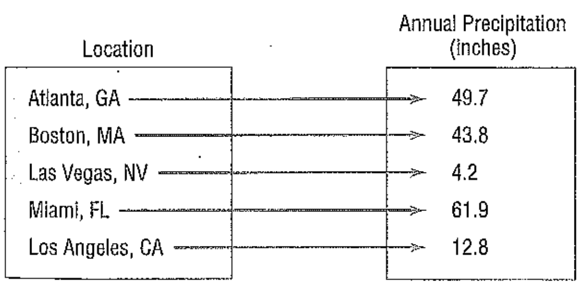


24.



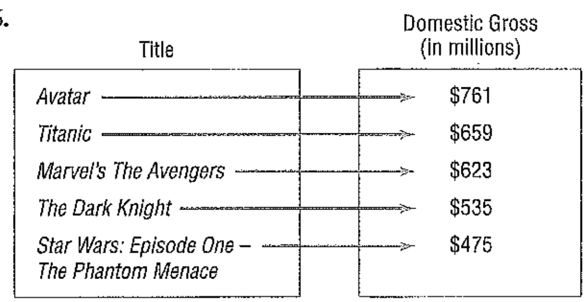
In Problems 25–32, find the inverse of each one-to-one function. State the domain and the range of each inverse function.

25.



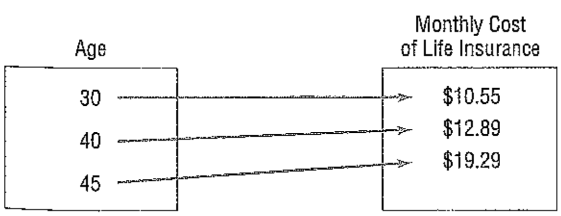
Source: currentresults.com

26.



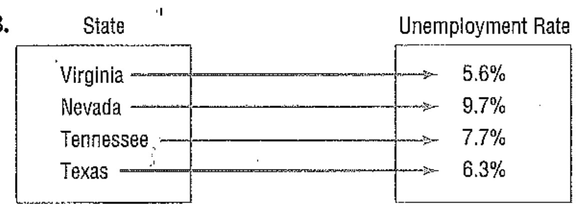
Source: boxofficemojo.com

27.



Source: acequotes.com

28.



Source: United States Bureau of Labor Statistics, Jan. 2013

29. $\{(-3, 5), (-2, 9), (-1, 2), (0, 11), (1, -5)\}$

30. $\{(-2, 2), (-1, 6), (0, 8), (1, -3), (2, 9)\}$

31. $\{(-2, 1), (-3, 2), (-10, 0), (1, 9), (2, 4)\}$

32. $\{(-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8)\}$

In Problems 33–42, verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded from the domain of f and the domain of g.

33. $f(x) = 3x + 4;\quad g(x) = \frac{1}{3}(x - 4)$

34. $f(x) = 3 - 2x;\quad g(x) = -\frac{1}{2}(x - 3)$

35. $f(x) = 4x - 8;\quad g(x) = \frac{x}{4} + 2$

36. $f(x) = 2x + 6;\quad g(x) = \frac{1}{2}x - 3$

37. $f(x) = x^3 - 8;\quad g(x) = \sqrt[3]{x + 8}$

38. $f(x) = (x - 2)^2, x \geq 2;\quad g(x) = \sqrt{x} + 2$

39. $f(x) = \dfrac{1}{x};\quad g(x) = \dfrac{1}{x}$

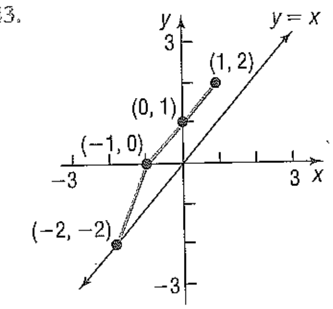
40. $f(x) = x;\quad g(x) = x$

41. $f(x) = \dfrac{2x + 3}{x + 4};\quad g(x) = \dfrac{4x - 3}{2 - x}$

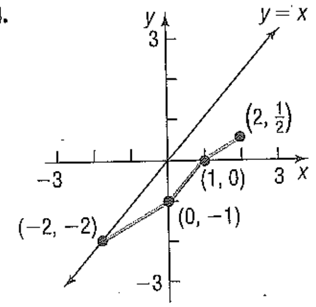
42. $f(x) = \dfrac{x - 5}{2x + 3};\quad g(x) = \dfrac{3x + 5}{1 - 2x}$

In Problems 43–48, the graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of $y = x$ is also given.

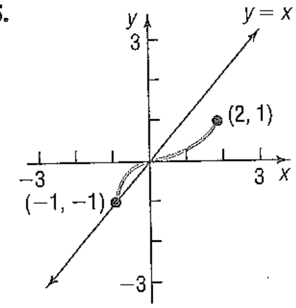
43.



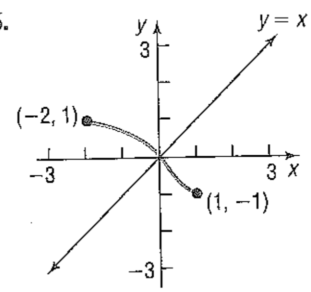
44.



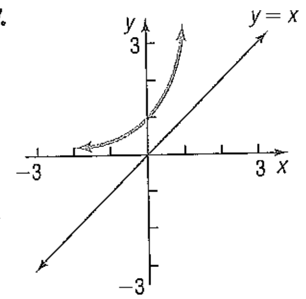
45.



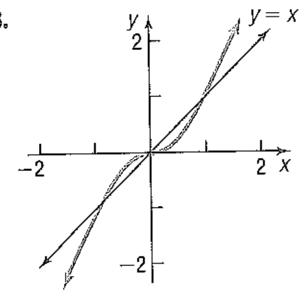
46.



47.



48.



In Problems 49–60, the function f is one-to-one. Find its inverse and check your answer. Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

49. $f(x) = 3x$

50. $f(x) = -4x$

51. $f(x) = 4x + 2$

52. $f(x) = 1 - 3x$

53. $f(x) = x^3 - 1$

54. $f(x) = x^3 + 1$

55. $f(x) = x^2 + 4,\quad x \geq 0$

56. $f(x) = x^2 + 9,\quad x \geq 0$

57. $f(x) = \dfrac{4}{x}$

58. $f(x) = -\dfrac{3}{x}$

59. $f(x) = \dfrac{1}{x - 2}$

60. $f(x) = \dfrac{4}{x + 2}$

In Problems 61–72, the function f is one-to-one. Find its inverse and check your answer.

61. $f(x) = \dfrac{2}{3 + x}$

62. $f(x) = \dfrac{4}{2 - x}$

63. $f(x) = \dfrac{3x}{x + 2}$

64. $f(x) = -\dfrac{2x}{x - 1}$

65. $f(x) = \dfrac{2x}{3x - 1}$

66. $f(x) = -\dfrac{3x + 1}{x}$

67. $f(x) = \dfrac{3x + 4}{2x - 3}$

68. $f(x) = \dfrac{2x - 3}{x + 4}$

69. $f(x) = \dfrac{2x + 3}{x + 2}$

70. $f(x) = \dfrac{-3x - 4}{x - 2}$

71. $f(x) = \dfrac{x^2 - 4}{2x^2},\quad x > 0$

72. $f(x) = \dfrac{x^2 + 3}{3x^2},\quad x > 0$

Applications and Extensions

73. Use the graph of $y = f(x)$ given in Problem 43 to evaluate the following:
(a) $f(-1)$ (b) $f(1)$ (c) $f^{-1}(1)$ (d) $f^{-1}(2)$

74. Use the graph of $y = f(x)$ given in Problem 44 to evaluate the following:
(a) $f(2)$ (b) $f(1)$ (c) $f^{-1}(0)$ (d) $f^{-1}(-1)$

75. If $f(7) = 13$ and f is one-to-one, what is $f^{-1}(13)$?

76. If $g(-5) = 3$ and g is one-to-one, what is $g^{-1}(3)$?

77. The domain of a one-to-one function f is $[5, \infty)$, and its range is $[-2, \infty)$. State the domain and the range of f^{-1}.

78. The domain of a one-to-one function f is $[0, \infty)$, and its range is $[5, \infty)$. State the domain and the range of f^{-1}.

79. The domain of a one-to-one function g is $(-\infty, 0]$, and its range is $[0, \infty)$. State the domain and the range of g^{-1}.

80. The domain of a one-to-one function g is $[0, 15]$, and its range is $(0, 8)$. State the domain and the range of g^{-1}.

81. A function $y = f(x)$ is increasing on the interval $(0, 5)$. What conclusions can you draw about the graph of $y = f^{-1}(x)$?

82. A function $y = f(x)$ is decreasing on the interval $(0, 5)$. What conclusions can you draw about the graph of $y = f^{-1}(x)$?

83. Find the inverse of the linear function

$$f(x) = mx + b, \quad m \neq 0$$

84. Find the inverse of the function

$$f(x) = \sqrt{r^2 - x^2}, \quad 0 \le x \le r$$

85. A function f has an inverse function. If the graph of f lies in quadrant I, in which quadrant does the graph of f^{-1} lie?

86. A function f has an inverse function. If the graph of f lies in quadrant II, in which quadrant does the graph of f^{-1} lie?

87. The function $f(x) = |x|$ is not one-to-one. Find a suitable restriction on the domain of f so that the new function that results is one-to-one. Then find the inverse of f.

88. The function $f(x) = x^4$ is not one-to-one. Find a suitable restriction on the domain of f so that the new function that results is one-to-one. Then find the inverse of f.

In applications, the symbols used for the independent and dependent variables are often based on common usage. So, rather than using $y = f(x)$ to represent a function, an applied problem might use $C = C(q)$ to represent the cost C of manufacturing q units of a good since, in economics, q is used for output. Because of this, the inverse notation f^{-1} used in a pure mathematics problem is not used when finding inverses of applied problems. Rather, the inverse of a function such as $C = C(q)$ will be $q = q(C)$. So $C = C(q)$ is a function that represents the cost C as a function of the output q, and $q = q(C)$ is a function that represents the output q as a function of the cost C. Problems 89–92 illustrate this idea.

89. **Vehicle Stopping Distance** Taking into account reaction time, the distance d (in feet) that a car requires to come to a complete stop while traveling r miles per hour is given by the function

$$d(r) = 6.97r - 90.39$$

(a) Express the speed r at which the car is traveling as a function of the distance d required to come to a complete stop.
(b) Verify that $r = r(d)$ is the inverse of $d = d(r)$ by showing that $r(d(r)) = r$ and $d(r(d)) = d$.
(c) Predict the speed that a car was traveling if the distance required to stop was 300 feet.

90. **Height and Head Circumference** The head circumference C of a child is related to the height H of the child (both in inches) through the function

$$H(C) = 2.15C - 10.53$$

(a) Express the head circumference C as a function of height H.
(b) Verify that $C = C(H)$ is the inverse of $H = H(C)$ by showing that $H(C(H)) = H$ and $C(H(C)) = C$.
(c) Predict the head circumference of a child who is 26 inches tall.

91. **Ideal Body Weight** One model for the ideal body weight W for men (in kilograms) as a function of height h (in inches) is given by the function

$$W(h) = 50 + 2.3(h - 60)$$

(a) What is the ideal weight of a 6-foot male?
(b) Express the height h as a function of weight W.
(c) Verify that $h = h(W)$ is the inverse of $W = W(h)$ by showing that $h(W(h)) = h$ and $W(h(W)) = W$.
(d) What is the height of a male who is at his ideal weight of 80 kilograms?

[**Note:** The ideal body weight W for women (in kilograms) as a function of height h (in inches) is given by $W(h) = 45.5 + 2.3(h - 60)$.]

92. **Temperature Conversion** The function $F(C) = \frac{9}{5}C + 32$ converts a temperature from C degrees Celsius to F degrees Fahrenheit.

(a) Express the temperature in degrees Celsius C as a function of the temperature in degrees Fahrenheit F.
(b) Verify that $C = C(F)$ is the inverse of $F = F(C)$ by showing that $C(F(C)) = C$ and $F(C(F)) = F$.
(c) What is the temperature in degrees Celsius if it is 70 degrees Fahrenheit?

93. **Income Taxes** The function

$$T(g) = 4991.25 + 0.25(g - 36{,}250)$$

represents the 2013 federal income tax T (in dollars) due for a "single" filer whose modified adjusted gross income is g dollars, where $36{,}250 \le g \le 87{,}850$.

(a) What is the domain of the function T?
(b) Given that the tax due T is an increasing linear function of modified adjusted gross income g, find the range of the function T.
(c) Find adjusted gross income g as a function of federal income tax T. What are the domain and the range of this function?

94. **Income Taxes** The function

$$T(g) = 1785 + 0.15(g - 17{,}850)$$

represents the 2013 federal income tax T (in dollars) due for a "married filing jointly" filer whose modified adjusted gross income is g dollars, where $17{,}850 \le g \le 72{,}500$.

(a) What is the domain of the function T?
(b) Given that the tax due T is an increasing linear function of modified adjusted gross income g, find the range of the function T.
(c) Find adjusted gross income g as a function of federal income tax T. What are the domain and the range of this function?

95. **Gravity on Earth** If a rock falls from a height of 100 meters on Earth, the height H (in meters) after t seconds is approximately

$$H(t) = 100 - 4.9t^2$$

(a) In general, quadratic functions are not one-to-one. However, the function H is one-to-one. Why?
(b) Find the inverse of H and verify your result.
(c) How long will it take a rock to fall 80 meters?

96. **Period of a Pendulum** The period T (in seconds) of a simple pendulum as a function of its length l (in feet) is given by

$$T(l) = 2\pi\sqrt{\frac{l}{32.2}}$$

(a) Express the length l as a function of the period T.
(b) How long is a pendulum whose period is 3 seconds?

97. Given

$$f(x) = \frac{ax + b}{cx + d}$$

find $f^{-1}(x)$. If $c \neq 0$, under what conditions on a, b, c, and d is $f = f^{-1}$?

Discussion and Writing

98. Can a one-to-one function and its inverse be equal? What must be true about the graph of f for this to happen? Give some examples to support your conclusion.
99. Draw the graph of a one-to-one function that contains the points $(-2, -3)$, $(0, 0)$, and $(1, 5)$. Now draw the graph of its inverse. Compare your graph to those of other students. Discuss any similarities. What differences do you see?
100. Give an example of a function whose domain is the set of real numbers and that is neither increasing nor decreasing on its domain, but is one-to-one.
[**Hint:** Use a piecewise-defined function.]
101. Is every odd function one-to-one? Explain.
102. Suppose that $C(g)$ represents the cost C, in dollars, of manufacturing g cars. Explain what $C^{-1}(800{,}000)$ represents.
103. Explain why the horizontal-line test can be used to identify one-to-one functions from a graph.
104. Explain why a function must be one-to-one in order to have an inverse that is a function. Use the function $y = x^2$ to support your explanation.

Retain Your Knowledge

Problems 105–108 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

105. If $f(x) = 3x^2 - 7x$, find $f(x + h) - f(x)$
106. Use the techniques of shifting, compressing or stretching, and reflections to graph $f(x) = -|x + 2| + 3$.
107. Find the zeros of the quadratic function $f(x) = 3x^2 + 5x + 1$. What are the x-intercepts, if any, of the graph of the function?
108. Find the domain of $R(x) = \dfrac{6x^2 - 11x - 2}{2x^2 - x - 6}$. Find any horizontal, vertical, or oblique asymptotes.

'Are You Prepared?' Answers

1. Yes; for each input x there is one output y.
2. Increasing on $(0, \infty)$; decreasing on $(-\infty, 0)$
3. $\{x \mid x \neq -6, x \neq 3\}$
4. $\dfrac{x}{1 - x}, x \neq 0, x \neq -1, x \neq 1$

4.3 Exponential Functions

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Exponents (Appendix A, Section A.1, pp. A7–A9, and Section A.7, pp. A58–A60)
- Graphing Techniques: Transformations (Section 1.5, pp. 89–97)
- Linear Equations (Appendix A, Section A.8, pp. A63–A69)
- Average Rate of Change (Section 1.3, pp. 72–74)
- Quadratic Functions (Section 2.3, pp. 137–145)
- Linear Functions (Section 2.1, pp. 119–123)
- Horizontal Asymptotes (Section 3.4, pp. 234–240)

Now Work the 'Are You Prepared?' problems on page 305.

OBJECTIVES
1 Evaluate Exponential Functions (p. 294)
2 Graph Exponential Functions (p. 298)
3 Define the Number e (p. 301)
4 Solve Exponential Equations (p. 303)

1 Evaluate Exponential Functions

Appendix A, Section A.7 gives a definition for raising a real number a to a rational power. That discussion provides meaning to expressions of the form