5.4 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Use transformations to graph $y = 3x^2$. (pp. 92–93)

2. Use transformations to graph $y = \sqrt{2x}$. (pp. 92–93)

Concepts and Vocabulary

3. The maximum value of $y = \sin x$, $0 \le x \le 2\pi$, is ____ and occurs at $x =$ ____.

4. The function $y = A\sin(\omega x)$, $A > 0$, has amplitude 3 and period 2; then $A =$ ____ and $\omega =$ ____.

5. The function $y = 3\cos(6x)$ has amplitude ____ and period ____.

6. ***True or False*** The graphs of $y = \sin x$ and $y = \cos x$ are identical except for a horizontal shift.

7. ***True or False*** For $y = 2\sin(\pi x)$, the amplitude is 2 and the period is $\frac{\pi}{2}$.

8. ***True or False*** The graph of the sine function has infinitely many x-intercepts.

Skill Building

9. $f(x) = \sin x$
- (a) What is the y-intercept of the graph of f?
- (b) For what numbers x, $-\pi \le x \le \pi$, is the graph of f increasing?
- (c) What is the absolute maximum of f?
- (d) For what numbers x, $0 \le x \le 2\pi$, does $f(x) = 0$?
- (e) For what numbers x, $-2\pi \le x \le 2\pi$, does $f(x) = 1$? Where does $f(x) = -1$?
- (f) For what numbers x, $-2\pi \le x \le 2\pi$, does $f(x) = -\frac{1}{2}$?
- (g) What are the x-intercepts of f?

10. $g(x) = \cos x$
- (a) What is the y-intercept of the graph of g?
- (b) For what numbers x, $-\pi \le x \le \pi$, is the graph of g decreasing?
- (c) What is the absolute minimum of g?
- (d) For what numbers x, $0 \le x \le 2\pi$, does $g(x) = 0$?
- (e) For what numbers x, $-2\pi \le x \le 2\pi$, does $g(x) = 1$? Where does $g(x) = -1$?
- (f) For what numbers x, $-2\pi \le x \le 2\pi$, does $g(x) = \frac{\sqrt{3}}{2}$?
- (g) What are the x-intercepts of g?

In Problems 11–20, determine the amplitude and period of each function without graphing.

11. $y = 2\sin x$

12. $y = 3\cos x$

13. $y = -4\cos(2x)$

14. $y = -\sin\left(\frac{1}{2}x\right)$

15. $y = 6\sin(\pi x)$

16. $y = -3\cos(3x)$

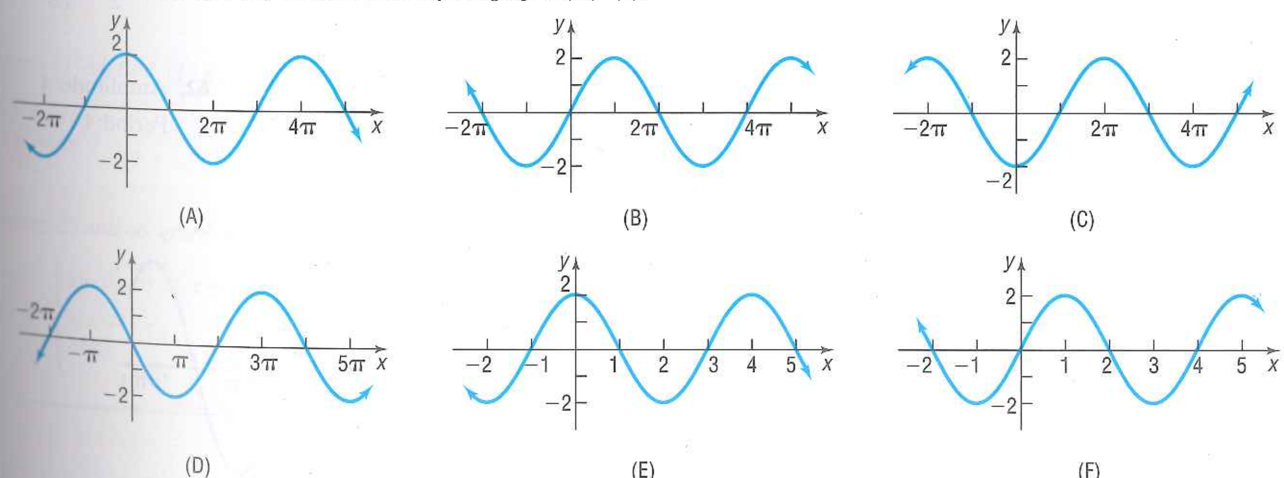
17. $y = -\frac{1}{2}\cos\left(\frac{3}{2}x\right)$

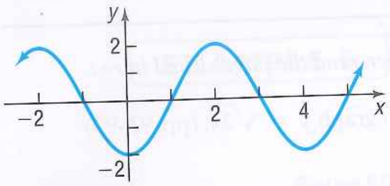
18. $y = \frac{4}{3}\sin\left(\frac{2}{3}x\right)$

19. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$

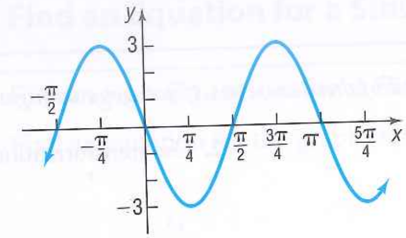
20. $y = \frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right)$

In Problems 21–30, match the given function to one of the graphs (A)–(J).

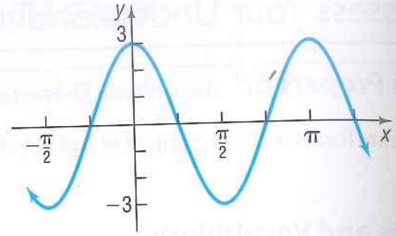




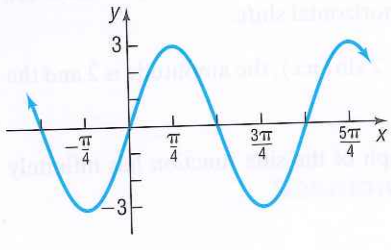
(G)



(H)



(I)



(J)

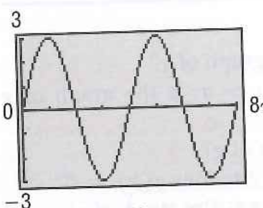
21. $y = 2\sin\left(\frac{\pi}{2}x\right)$ **22.** $y = 2\cos\left(\frac{\pi}{2}x\right)$ **23.** $y = 2\cos\left(\frac{1}{2}x\right)$

24. $y = 3\cos(2x)$ **25.** $y = -3\sin(2x)$ **26.** $y = 2\sin\left(\frac{1}{2}x\right)$

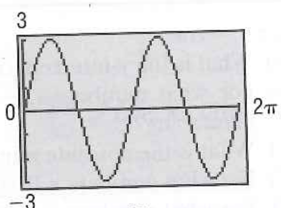
27. $y = -2\cos\left(\frac{1}{2}x\right)$ **28.** $y = -2\cos\left(\frac{\pi}{2}x\right)$ **29.** $y = 3\sin(2x)$

30. $y = -2\sin\left(\frac{1}{2}x\right)$

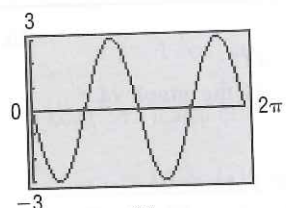
In Problems 31–34, match the given function to one of the graphs (A)–(D).



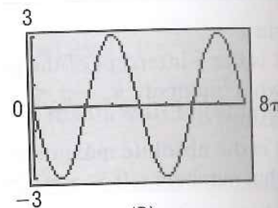
(A)



(B)



(C)



(D)

31. $y = 3\sin\left(\frac{1}{2}x\right)$ **32.** $y = -3\sin(2x)$ **33.** $y = 3\sin(2x)$ **34.** $y = -3\sin\left(\frac{1}{2}x\right)$

In Problems 35–58, graph each function. Be sure to label key points and show at least two cycles. Use the graph to determine the domain and the range of each function.

35. $y = 4\cos x$ **36.** $y = 3\sin x$ **37.** $y = -4\sin x$ **38.** $y = -3\cos x$

39. $y = \cos(4x)$ **40.** $y = \sin(3x)$ **41.** $y = \sin(-2x)$ **42.** $y = \cos(-2x)$

43. $y = 2\sin\left(\frac{1}{2}x\right)$ **44.** $y = 2\cos\left(\frac{1}{4}x\right)$ **45.** $y = -\frac{1}{2}\cos(2x)$ **46.** $y = -4\sin\left(\frac{1}{8}x\right)$

47. $y = 2\sin x + 3$ **48.** $y = 3\cos x + 2$ **49.** $y = 5\cos(\pi x) - 3$ **50.** $y = 4\sin\left(\frac{\pi}{2}x\right) - 2$

51. $y = -6\sin\left(\frac{\pi}{3}x\right) + 4$ **52.** $y = -3\cos\left(\frac{\pi}{4}x\right) + 2$ **53.** $y = 5 - 3\sin(2x)$ **54.** $y = 2 - 4\cos(3x)$

55. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$ **56.** $y = \frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right)$ **57.** $y = -\frac{3}{2}\cos\left(\frac{\pi}{4}x\right) + \frac{1}{2}$ **58.** $y = -\frac{1}{2}\sin\left(\frac{\pi}{8}x\right) + \frac{3}{2}$

In Problems 59–62, write the equation of a sine function that has the given characteristics.

59. Amplitude: 3
Period: π

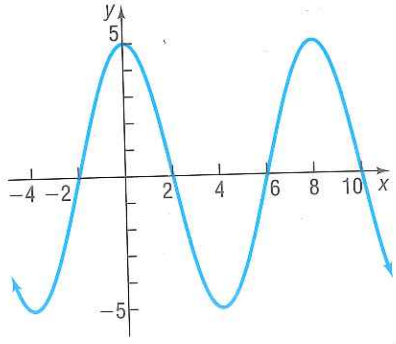
60. Amplitude: 2
Period: 4π

61. Amplitude: 3
Period: 2

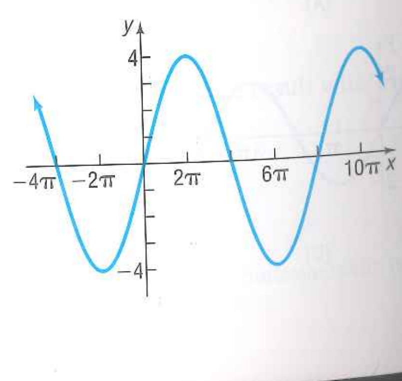
62. Amplitude: 4
Period: 1

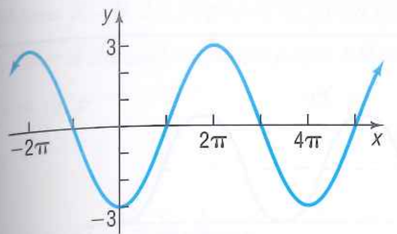
In Problems 63–76, find an equation for each graph.

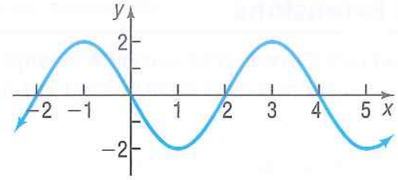
63.

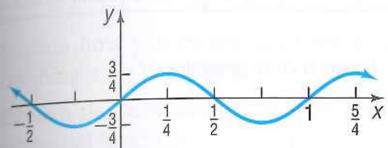


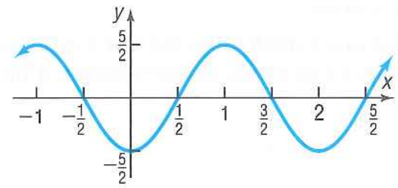
64.

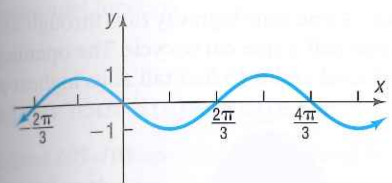


65.


66.


67.


68.


69.


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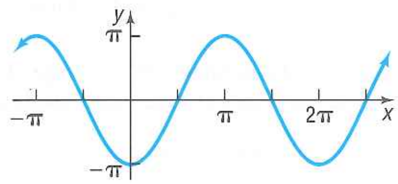
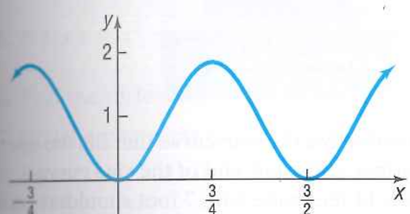
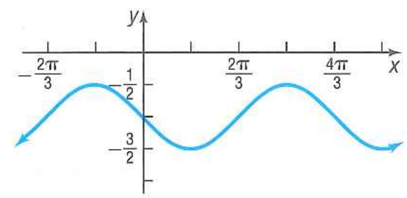
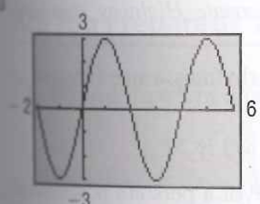


71.


72.


73.


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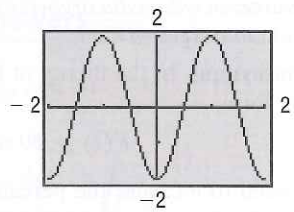
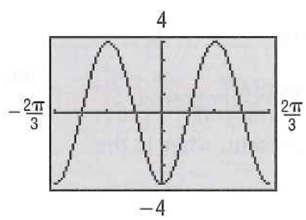
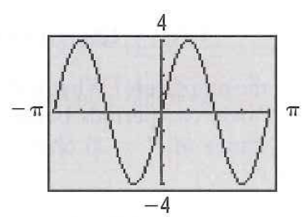


75.


76.


Mixed Practice

In Problems 77–80, find the average rate of change of f from 0 to $\frac{\pi}{2}$.

77. $f(x) = \sin x$ **78.** $f(x) = \cos x$ **79.** $f(x) = \sin\left(\frac{x}{2}\right)$ **80.** $f(x) = \cos(2x)$

In Problems 81–84, find $(f \circ g)(x)$ and $(g \circ f)(x)$, and graph each of these functions.

81. $f(x) = \sin x$, $g(x) = 4x$

82. $f(x) = \cos x$, $g(x) = \frac{1}{2}x$

83. $f(x) = -2x$, $g(x) = \cos x$

84. $f(x) = -3x$, $g(x) = \sin x$

In Problems 85 and 86, graph each function.

85. $f(x) = \begin{cases} \sin x & 0 \le x < \frac{5\pi}{4} \\ \cos x & \frac{5\pi}{4} \le x \le 2\pi \end{cases}$

86. $g(x) = \begin{cases} 2\sin x & 0 \le x \le \pi \\ \cos x + 1 & \pi < x \le 2\pi \end{cases}$

Applications and Extensions

87. **Alternating Current (ac) Circuits** The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 220 \sin(60\pi t) \qquad t \geq 0$$

What is the period? What is the amplitude? Graph this function over two periods.

88. **Alternating Current (ac) Circuits** The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 120 \sin(30\pi t) \qquad t \geq 0$$

What is the period? What is the amplitude? Graph this function over two periods.

89. **Alternating Current (ac) Generators** The voltage V, in volts, produced by an ac generator at time t, in seconds, is

$$V(t) = 220 \sin(120\pi t)$$

(a) What is the amplitude? What is the period?
(b) Graph V over two periods, beginning at $t = 0$.
(c) If a resistance of $R = 10$ ohms is present, what is the current I?

[**Hint:** Use Ohm's Law, $V = IR$.]

(d) What are the amplitude and period of the current I?
(e) Graph I over two periods, beginning at $t = 0$.

90. **Alternating Current (ac) Generators** The voltage V, in volts, produced by an ac generator at time t, in seconds, is

$$V(t) = 120 \sin(120\pi t)$$

(a) What is the amplitude? What is the period?
(b) Graph V over two periods, beginning at $t = 0$.
(c) If a resistance of $R = 20$ ohms is present, what is the current I?

[**Hint:** Use Ohm's Law, $V = IR$.]

(d) What are the amplitude and period of the current I?
(e) Graph I over two periods, beginning at $t = 0$.

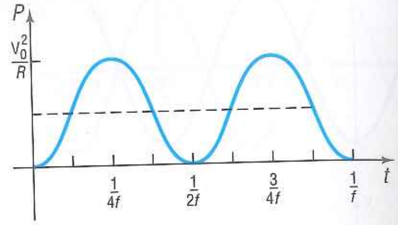
91. **Alternating Current (ac) Generators** The voltage V produced by an ac generator is sinusoidal. As a function of time, the voltage V is

$$V(t) = V_0 \sin(2\pi f t)$$

where f is the **frequency,** the number of complete oscillations (cycles) per second, and V_0 is the initial voltage. [In the United States and Canada, f is 60 hertz (Hz).] The **power** P delivered to a resistance R at any time t is defined as

$$P(t) = \frac{[V(t)]^2}{R}$$

(a) Show that $P(t) = \frac{V_0^2}{R} \sin^2(2\pi f t)$.
(b) The graph of P is shown in the figure. Express P as a sinusoidal function.

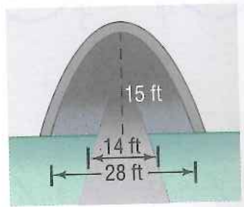


Power in an ac generator

(c) Deduce that

$$\sin^2(2\pi f t) = \frac{1}{2}[1 - \cos(4\pi f t)]$$

92. **Bridge Clearance** A one-lane highway runs through a tunnel in the shape of one-half a sine curve cycle. The opening is 28 feet wide at road level and is 15 feet tall at its highest point.



(a) Find an equation for the sine curve that fits the opening. Place the origin at the left end of the sine curve.
(b) If the road is 14 feet wide with 7-foot shoulders on each side, what is the height of the tunnel at the edge of the road?

Source: en.wikipedia.org/wiki/Interstate_Highway_standards and Ohio Revised Code

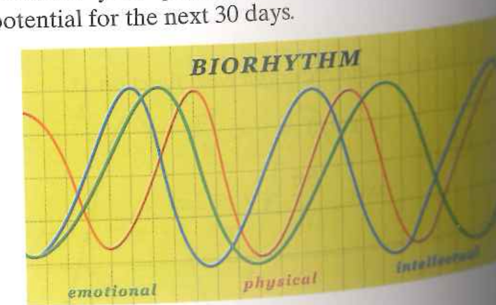
93. **Biorhythms** In the theory of biorhythms, a sine function of the form

$$P(t) = 50 \sin(\omega t) + 50$$

is used to measure the percent P of a person's potential at time t, where t is measured in days and $t = 0$ is the person's birthday. Three characteristics are commonly measured:

Physical potential: period of 23 days
Emotional potential: period of 28 days
Intellectual potential: period of 33 days

(a) Find ω for each characteristic.
(b) Using a graphing utility, graph all three functions on the same screen.
(c) Is there a time t when all three characteristics have 100% potential? When is it?
(d) Suppose that you are 20 years old today ($t = 7305$ days). Describe your physical, emotional, and intellectual potential for the next 30 days.



94. Graph $y = |\cos x|, -2\pi \leq x \leq 2\pi$.

95. Graph $y = |\sin x|, -2\pi \leq x \leq 2\pi$.

In Problems 96–99, the graphs of the given pairs of functions intersect infinitely many times. Find four of these points of intersection.

[**Hint:** *Refer to the unit circles on pages 398 and 399, and use their periodic properties.*]

96. $y = \sin x$
$y = \frac{1}{2}$

97. $y = \cos x$
$y = \frac{1}{2}$

98. $y = 2 \sin x$
$y = -2$

99. $y = \tan x$
$y = 1$

Discussion and Writing

100. Explain how you would scale the x-axis and y-axis before graphing $y = 3\cos(\pi x)$.

101. Explain the term *amplitude* as it relates to the graph of a sinusoidal function.

102. Explain the term *period* as it relates to the graph of a sinusoidal function.

103. Explain how the amplitude and period of a sinusoidal graph are used to establish the scale on each coordinate axis.

104. Find an application in your major field that leads to a sinusoidal graph. Write an account of your findings.

Retain Your Knowledge

Problems 105–108 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

105. If $f(x) = x^2 - 5x + 1$, find $\dfrac{f(x+h) - f(x)}{h}$.

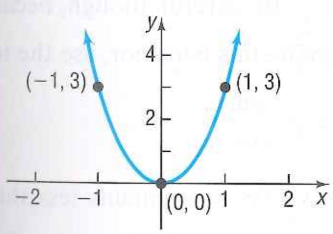
106. Find the vertex of the graph of $g(x) = -3x^2 + 12x - 7$.

107. Find the intercepts of the graph of $h(x) = 3|x + 2| - 1$.

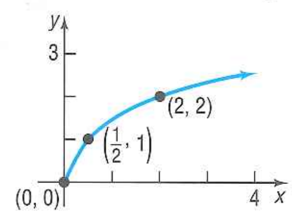
108. Solve: $3x - 2(5x + 16) = -3x + 4(8 - x)$

'Are You Prepared?' Answers

1. Vertical stretch by a factor of 3



2. Horizontal compression by a factor of $\frac{1}{2}$



5.5 Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Vertical Asymptotes (Section 3.4, pp. 234–237)

Now Work the **'Are You Prepared?'** problems on page 441.

OBJECTIVES
1. Graph Functions of the Form $y = A\tan(\omega x) + B$ and $y = A\cot(\omega x) + B$ (p. 437)
2. Graph Functions of the Form $y = A\csc(\omega x) + B$ and $y = A\sec(\omega x) + B$ (p. 440)