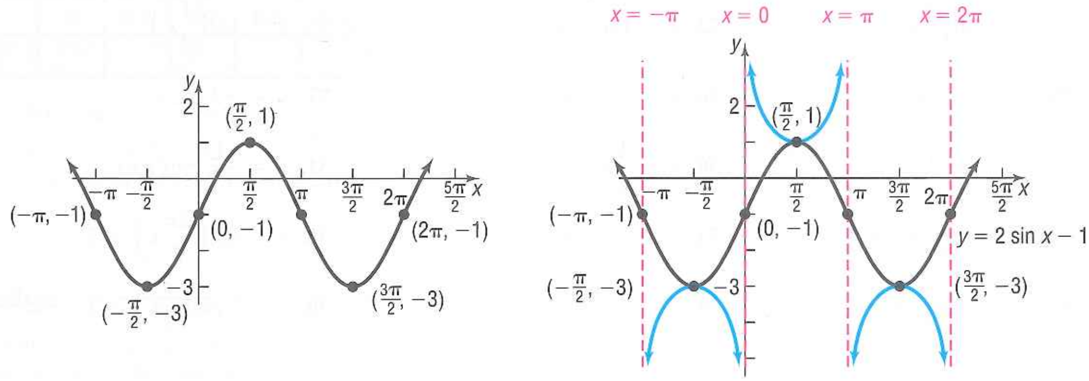
Solution Using the Reciprocal Function Graph $y = 2\csc x - 1$ by first graphing the function $y = 2\sin x - 1$ and then filling in the graph of $y = 2\csc x - 1$, using the idea of reciprocals. See Figure 70.

Figure 70



(a) $y = 2\sin x - 1$ (b) $y = 2\csc x - 1$

The domain of $y = 2\csc x - 1$ is $\{x | x \neq k\pi, k \text{ is an integer}\}$, and the range is $\{y | y \leq -3 \text{ or } y \geq 1\}$ or, using interval notation, $(-\infty, -3] \cup [1, \infty)$.

✓**Check:** Graph $Y_1 = 2\csc x - 1$ to verify the graph shown in Figure 69 or 70. ●

Now Work PROBLEM 29

5.5 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. The graph of $y = \dfrac{3x - 6}{x - 4}$ has a vertical asymptote. What is it? (pp. 234–237)

2. ***True or False*** If $x = 3$ is a vertical asymptote of a rational function R, then $\lim_{x \to 3} |R(x)| = \infty$. (pp. 234–237)

Concepts and Vocabulary

3. The graph of $y = \tan x$ is symmetric with respect to the ________ and has vertical asymptotes at ________.

4. The graph of $y = \sec x$ is symmetric with respect to the ______ and has vertical asymptotes at ________.

5. It is easiest to graph $y = \sec x$ by first sketching the graph of ________.

6. ***True or False*** The graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$, and $y = \csc x$ have infinitely many vertical asymptotes.

Skill Building

In Problems 7–16, if necessary, refer to the graphs to answer each question.

7. What is the y-intercept of $y = \tan x$?

8. What is the y-intercept of $y = \cot x$?

9. What is the y-intercept of $y = \sec x$?

10. What is the y-intercept of $y = \csc x$?

11. For what numbers x, $-2\pi \leq x \leq 2\pi$, does $\sec x = 1$? For what numbers x does $\sec x = -1$?

12. For what numbers x, $-2\pi \leq x \leq 2\pi$, does $\csc x = 1$? For what numbers x does $\csc x = -1$?

13. For what numbers x, $-2\pi \leq x \leq 2\pi$, does the graph of $y = \sec x$ have vertical asymptotes?

14. For what numbers x, $-2\pi \leq x \leq 2\pi$, does the graph of $y = \csc x$ have vertical asymptotes?

15. For what numbers x, $-2\pi \leq x \leq 2\pi$, does the graph of $y = \tan x$ have vertical asymptotes?

16. For what numbers x, $-2\pi \leq x \leq 2\pi$, does the graph of $y = \cot x$ have vertical asymptotes?

In Problems 17–40, graph each function. Be sure to label key points and show at least two cycles. Use the graph to determine the domain and the range of each function.

17. $y = 3\tan x$
18. $y = -2\tan x$
19. $y = 4\cot x$
20. $y = -3\cot x$
21. $y = \tan\left(\frac{\pi}{2}x\right)$
22. $y = \tan\left(\frac{1}{2}x\right)$
23. $y = \cot\left(\frac{1}{4}x\right)$
24. $y = \cot\left(\frac{\pi}{4}x\right)$
25. $y = 2\sec x$
26. $y = \frac{1}{2}\csc x$
27. $y = -3\csc x$
28. $y = -4\sec x$
29. $y = 4\sec\left(\frac{1}{2}x\right)$
30. $y = \frac{1}{2}\csc(2x)$
31. $y = -2\csc(\pi x)$
32. $y = -3\sec\left(\frac{\pi}{2}x\right)$
33. $y = \tan\left(\frac{1}{4}x\right) + 1$
34. $y = 2\cot x - 1$
35. $y = \sec\left(\frac{2\pi}{3}x\right) + 2$
36. $y = \csc\left(\frac{3\pi}{2}x\right)$
37. $y = \frac{1}{2}\tan\left(\frac{1}{4}x\right) - 2$
38. $y = 3\cot\left(\frac{1}{2}x\right) - 2$
39. $y = 2\csc\left(\frac{1}{3}x\right) - 1$
40. $y = 3\sec\left(\frac{1}{4}x\right) + 1$

Mixed Practice

In Problems 41–44, find the average rate of change of f from 0 to $\frac{\pi}{6}$.

41. $f(x) = \tan x$
42. $f(x) = \sec x$
43. $f(x) = \tan(2x)$
44. $f(x) = \sec(2x)$

In Problems 45–48, find $(f \circ g)(x)$ and $(g \circ f)(x)$, and graph each of these functions.

45. $f(x) = \tan x$, $g(x) = 4x$
46. $f(x) = 2\sec x$, $g(x) = \frac{1}{2}x$
47. $f(x) = -2x$, $g(x) = \cot x$
48. $f(x) = \frac{1}{2}x$, $g(x) = 2\csc x$

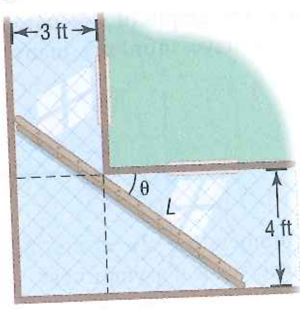
In Problems 49 and 50, graph each function.

49. $f(x) = \begin{cases} \tan x & 0 \le x < \frac{\pi}{2} \\ 0 & x = \frac{\pi}{2} \\ \sec x & \frac{\pi}{2} < x \le \pi \end{cases}$

50. $g(x) = \begin{cases} \csc x & 0 < x < \pi \\ 0 & x = \pi \\ \cot x & \pi < x < 2\pi \end{cases}$

Applications and Extensions

51. Carrying a Ladder around a Corner Two hallways, one of width 3 feet, the other of width 4 feet, meet at a right angle. See the illustration.

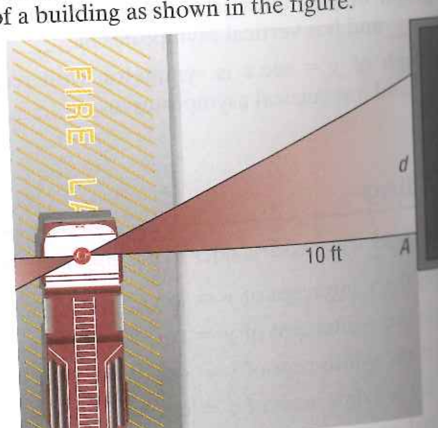


(a) Show that the length L of the line segment shown, as a function of the angle θ, is

$$L(\theta) = 3\sec\theta + 4\csc\theta$$

(b) Graph $L = L(\theta), 0 < \theta < \frac{\pi}{2}$.
(c) For what value of θ is L the least?
(d) What is the length of the longest ladder that can be carried around the corner? Why is this also the least value of L?

52. A Rotating Beacon Suppose that a fire truck is parked in front of a building as shown in the figure.



The beacon light on top of the fire truck is located 10 feet from the wall and has a light on each side. If the beacon light rotates 1 revolution every 2 seconds, then a model for determining the distance d, in feet, that the beacon of light is from point A on the wall after t seconds is given by

$$d(t) = |10\tan(\pi t)|$$

(a) Graph $d(t) = |10\tan(\pi t)|$ for $0 \le t \le 2$.

(b) For what values of t is the function undefined? Explain what this means in terms of the beam of light on the wall.

(c) Fill in the following table.

t	0	0.1	0.2	0.3	0.4
$d(t) = \lvert 10\tan(\pi t)\rvert$					

(d) Compute $\dfrac{d(0.1) - d(0)}{0.1 - 0}$, $\dfrac{d(0.2) - d(0.1)}{0.2 - 0.1}$, and so on, for each consecutive value of t. These are called **first differences.**

(e) Interpret the first differences found in part (d). What is happening to the speed of the beam of light as d increases?

53. Exploration Graph

$$y = \tan x \quad \text{and} \quad y = -\cot\left(x + \frac{\pi}{2}\right)$$

Do you think that $\tan x = -\cot\left(x + \dfrac{\pi}{2}\right)$?

Retain Your Knowledge

Problems 54–57 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

54. Factor: $125p^3 - 8q^6$

55. Painting a Room Hazel can paint a room in 2 hours less time than her friend Gwyneth. Working together, they can paint the room in 2.4 hours. How long does it take each friend to paint the room by herself?

56. Solve: $9^{x-1} = 3^{x^2-5}$

57. Use the slope and the y-intercept to graph the linear function $f(x) = \dfrac{1}{4}x - 3$.

'Are You Prepared?' Answers

1. $x = 4$ **2.** True

5.6 Phase Shift; Sinusoidal Curve Fitting

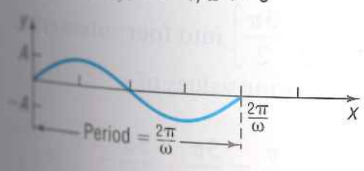
OBJECTIVES **1** Graph Sinusoidal Functions of the Form $y = A\sin(\omega x - \phi) + B$ (p. 443)

2 Build Sinusoidal Models from Data (p. 447)

1 Graph Sinusoidal Functions of the Form $y = A\sin(\omega x - \phi) + B$

We have seen that the graph of $y = A\sin(\omega x)$, $\omega > 0$, has amplitude $|A|$ and period $T = \dfrac{2\pi}{\omega}$. One cycle can be drawn as x varies from 0 to $\dfrac{2\pi}{\omega}$ or, equivalently, as ωx varies from 0 to 2π. See Figure 71.

Figure 71 One cycle of $y = A\sin(\omega x)$, $A > 0$, $\omega > 0$



Now consider the graph of

$$y = A\sin(\omega x - \phi)$$

which may also be written as

$$y = A\sin\left[\omega\left(x - \frac{\phi}{\omega}\right)\right]$$

where $\omega > 0$ and ϕ (the Greek letter phi) are real numbers. The graph is a sine curve with amplitude $|A|$. As $\omega x - \phi$ varies from 0 to 2π, one period is traced out. This period begins when

$$\omega x - \phi = 0 \quad \text{or} \quad x = \frac{\phi}{\omega}$$

NOTE The beginning and end of the period can also be found by solving the inequality

$$0 \le \omega x - \phi \le 2\pi$$
$$\phi \le \omega x \le 2\pi + \phi$$
$$\frac{\phi}{\omega} \le x \le \frac{2\pi}{\omega} + \frac{\phi}{\omega}$$ ■