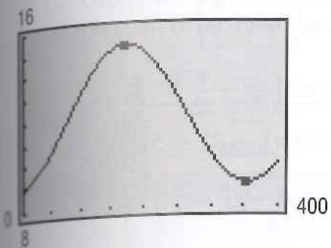
Figure 83



(b) To predict the number of hours of daylight on April 1, let $x = 91$ in the function found in part (a) and obtain

$$y = 3.11 \sin\left(\frac{2\pi}{365}\cdot 91 - \frac{323}{730}\pi\right) + 12.19$$
$$\approx 12.74$$

The prediction is that there will be about 12.74 hours = 12 hours, 44 minutes of sunlight on April 1 in Boston.

(c) The graph of the function found in part (a) is given in Figure 83.

(d) According to the *Old Farmer's Almanac,* there will be 12 hours 45 minutes of sunlight on April 1 in Boston.

Now Work PROBLEM 35

Certain graphing utilities (such as a TI-83, TI-84 Plus, and TI-86) have the capability of finding the sine function of best fit for sinusoidal data. At least four data points are required for this process.

EXAMPLE 5 Finding the Sine Function of Best Fit

Use a graphing utility to find the sine function of best fit for the data in Table 11. Graph this function with the scatter diagram of the data.

Solution Enter the data from Table 11 and execute the SINe REGression program. The result is shown in Figure 84.

The output that the utility provides shows the equation

$$y = a\sin(bx + c) + d$$

The sinusoidal function of best fit is

$$y = 21.43\sin(0.56x - 2.44) + 51.71$$

where x represents the month and y represents the average temperature.

Figure 85 shows the graph of the sinusoidal function of best fit on the scatter diagram.

Figure 84

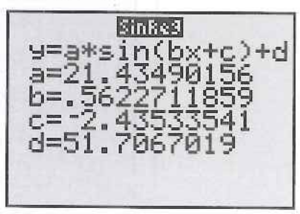
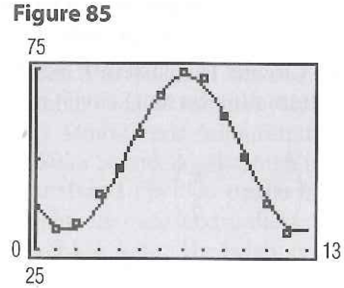


Figure 85



Now Work PROBLEMS 29(d) AND (e)

5.6 Assess Your Understanding

Concepts and Vocabulary

1. For the graph of $y = A\sin(\omega x - \phi)$, the number $\frac{\phi}{\omega}$ is called the _____ _____.

2. ***True or False*** A graphing utility requires only two data points to find the sine function of best fit.

Skill Building

In Problems 3–14, find the amplitude, period, and phase shift of each function. Graph each function. Be sure to label key points. Show at least two periods.

3. $y = 4\sin(2x - \pi)$

4. $y = 3\sin(3x - \pi)$

5. $y = 2\cos\left(3x + \frac{\pi}{2}\right)$

6. $y = 3\cos(2x + \pi)$

7. $y = -3\sin\left(2x + \frac{\pi}{2}\right)$

8. $y = -2\cos\left(2x - \frac{\pi}{2}\right)$

9. $y = 4\sin(\pi x + 2) - 5$

10. $y = 2\cos(2\pi x + 4) + 4$

11. $y = 3\cos(\pi x - 2) + 5$

12. $y = 2\cos(2\pi x - 4) - 1$

13. $y = -3\sin\left(-2x + \frac{\pi}{2}\right)$

14. $y = -3\cos\left(-2x + \frac{\pi}{2}\right)$

In Problems 15–18, write the equation of a sine function that has the given characteristics.

15. Amplitude: 2
Period: π
Phase shift: $\frac{1}{2}$

16. Amplitude: 3
Period: $\frac{\pi}{2}$
Phase shift: 2

17. Amplitude: 3
Period: 3π
Phase shift: $-\frac{1}{3}$

18. Amplitude: 2
Period: π
Phase shift: -2

Mixed Practice

In Problems 19–26, apply the methods of this and the previous section to graph each function. Be sure to label key points and show at least two periods.

19. $y = 2\tan(4x - \pi)$

20. $y = \frac{1}{2}\cot(2x - \pi)$

21. $y = 3\csc\left(2x - \frac{\pi}{4}\right)$

22. $y = \frac{1}{2}\sec(3x - \pi)$

23. $y = -\cot\left(2x + \frac{\pi}{2}\right)$

24. $y = -\tan\left(3x + \frac{\pi}{2}\right)$

25. $y = -\sec(2\pi x + \pi)$

26. $y = -\csc\left(-\frac{1}{2}\pi x + \frac{\pi}{4}\right)$

Applications and Extensions

27. Alternating Current (ac) Circuits The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 120\sin\left(30\pi t - \frac{\pi}{3}\right) \qquad t \geq 0$$

What is the period? What is the amplitude? What is the phase shift? Graph this function over two periods.

28. Alternating Current (ac) Circuits The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 220\sin\left(60\pi t - \frac{\pi}{6}\right) \qquad t \geq 0$$

What is the period? What is the amplitude? What is the phase shift? Graph this function over two periods.

29. Hurricanes Hurricanes are categorized using the Saffir-Simpson Hurricane Scale, with winds 111–130 miles per hour (mph) corresponding to a category 3 hurricane, winds 131–155 mph corresponding to a category 4 hurricane, and winds in excess of 155 mph corresponding to a category 5 hurricane. The following data represent the number of major hurricanes in the Atlantic Basin (category 3, 4, or 5) each decade from 1921 to 2010.

(a) Draw a scatter diagram of the data.

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(c) Draw the sinusoidal function found in part (b) on the scatter diagram.

(d) Use a graphing utility to find the sinusoidal function of best fit.

(e) Draw the sinusoidal function of best fit on a scatter diagram of the data.

Decade, x	Major Hurricanes, H
1921–1930, 1	17
1931–1940, 2	16
1941–1950, 3	29
1951–1960, 4	33
1961–1970, 5	27
1971–1980, 6	16
1981–1990, 7	16
1991–2000, 8	27
2001–2010, 9	33

Source: U.S. National Oceanic and Atmospheric Administration

30. Monthly Temperature The data on the next page represent the average monthly temperatures for Washington, D.C.

(a) Draw a scatter diagram of the data for one period.

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(c) Draw the sinusoidal function found in part (b) on the scatter diagram.

(d) Use a graphing utility to find the sinusoidal function of best fit.

(e) Graph the sinusoidal function of best fit on a scatter diagram of the data.

Month, x	Average Monthly Temperature, °F
January, 1	36.0
February, 2	39.0
March, 3	46.8
April, 4	56.8
May, 5	66.0
June, 6	75.2
July, 7	79.8
August, 8	78.1
September, 9	71.0
October, 10	59.5
November, 11	49.6
December, 12	39.7

Source: U.S. National Oceanic and Atmospheric Administration

31. Monthly Temperature The following data represent the average monthly temperatures for Indianapolis, Indiana.

Month, x	Average Monthly Temperature, °F
January, 1	28.1
February, 2	32.1
March, 3	42.2
April, 4	53.0
May, 5	62.7
June, 6	72.0
July, 7	75.4
August, 8	74.2
September, 9	66.9
October, 10	55.0
November, 11	43.6
December, 12	31.6

Source: U.S. National Oceanic and Atmospheric Administration

(a) Draw a scatter diagram of the data for one period.

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(c) Draw the sinusoidal function found in part (b) on the scatter diagram.

(d) Use a graphing utility to find the sinusoidal function of best fit.

(e) Graph the sinusoidal function of best fit on a scatter diagram of the data.

32. Monthly Temperature The following data represent the average monthly temperatures for Baltimore, Maryland.

(a) Draw a scatter diagram of the data for one period.

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(c) Draw the sinusoidal function found in part (b) on the scatter diagram.

(d) Use a graphing utility to find the sinusoidal function of best fit.

(e) Graph the sinusoidal function of best fit on a scatter diagram of the data.

Month, x	Average Monthly Temperature, °F
January, 1	32.9
February, 2	35.8
March, 3	43.6
April, 4	53.7
May, 5	62.9
June, 6	72.4
July, 7	77.0
August, 8	75.1
September, 9	67.8
October, 10	56.1
November, 11	46.5
December, 12	36.7

Source: U.S. National Oceanic and Atmospheric Administration

33. Tides The length of time between consecutive high tides is 12 hours and 25 minutes. According to the National Oceanic and Atmospheric Administration, on Saturday, July 21, 2012, in Charleston, South Carolina, high tide occurred at 11:30 AM (11.5 hours) and low tide occurred at 5:31 PM (17.5167 hours). Water heights are measured as the amounts above or below the mean lower low water. The height of the water at high tide was 5.84 feet, and the height of the water at low tide was −0.37 foot.

(a) Approximately when will the next high tide occur?

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(c) Use the function found in part (b) to predict the height of the water at 3 PM on July 21, 2012.

34. Tides The length of time between consecutive high tides is 12 hours and 25 minutes. According to the National Oceanic and Atmospheric Administration, on Saturday, July 21, 2012, in Sitka Sound, Alaska, high tide occurred at 2:37 AM (2.6167 hours) and low tide occurred at 9:12 PM (9.2 hours). Water heights are measured as the amounts above or below the mean lower low water. The height of the water at high tide was 11.09 feet, and the height of the water at low tide was −2.49 feet.

(a) Approximately when will the next high tide occur?

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(c) Use the function found in part (b) to predict the height of the water at 6 PM.

35. Hours of Daylight According to the *Old Farmer's Almanac*, in Miami, Florida, the number of hours of sunlight on the summer solstice of 2013 was 13.75, and the number of hours of sunlight on the winter solstice was 10.52.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac*, and compare the actual hours of daylight to the results found in part (b).

36. Hours of Daylight According to the *Old Farmer's Almanac*, in Detroit, Michigan, the number of hours of sunlight on the summer solstice of 2013 was 15.27, and the number of hours of sunlight on the winter solstice was 9.07.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac*, and compare the actual hours of daylight to the results found in part (b).

37. Hours of Daylight According to the *Old Farmer's Almanac*, in Anchorage, Alaska, the number of hours of sunlight on the summer solstice of 2013 was 19.37, and the number of hours of sunlight on the winter solstice was 5.45.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac*, and compare the actual hours of daylight to the results found in part (b).

38. Hours of Daylight According to the *Old Farmer's Almanac*, in Honolulu, Hawaii, the number of hours of sunlight on the summer solstice of 2013 was 13.42, and the number of hours of sunlight on the winter solstice was 10.83.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that models the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac*, and compare the actual hours of daylight to the results found in part (b).

Discussion and Writing

39. Explain how the amplitude and period of a sinusoidal graph are used to establish the scale on each coordinate axis.

40. Find an application in your major field that leads to a sinusoidal graph. Write an account of your findings.

Retain Your Knowledge

Problems 41–44 are based on material learned earlier in the course. The purpose of these problems to keep the material fresh in your mind so that you are better prepared for the final exam.

41. Given $f(x) = \dfrac{4x+9}{2}$, find $f^{-1}(x)$.

42. Solve: $0.25(0.4x + 0.8) = 3.7 - 1.4x$

43. Multiply: $(8x + 15y)^2$

44. Find the exact distance between the points $(4, -1)$ and $(10, 3)$.

Chapter Review

Things to Know

Definitions

Angle in standard position (p. 376) — Vertex is at the origin; initial side is along the positive x-axis.

1 Degree (1°) (p. 377) — $1^\circ = \dfrac{1}{360}$ revolution

1 Radian (p. 379) — The measure of a central angle of a circle whose rays subtend an arc that is the same length as the radius of the circle.

Trigonometric functions (pp. 390–393) — $P = (x, y)$ is the point on the unit circle corresponding to $\theta = t$ radians.

$\sin t = \sin\theta = y$ $\quad \cos t = \cos\theta = x$ $\quad \tan t = \tan\theta = \dfrac{y}{x}$ $\; x \neq 0$

$\csc t = \csc\theta = \dfrac{1}{y}$ $\; y \neq 0$ $\quad \sec t = \sec\theta = \dfrac{1}{x}$ $\; x \neq 0$ $\quad \cot t = \cot\theta = \dfrac{x}{y}$ $\; y \neq 0$

Trigonometric functions using a circle of radius r (p. 401) — For an angle θ in standard position, $P = (x, y)$ is the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$.

$\sin\theta = \dfrac{y}{r}$ $\quad \cos\theta = \dfrac{x}{r}$ $\quad \tan\theta = \dfrac{y}{x}$ $\; x \neq 0$

$\csc\theta = \dfrac{r}{y}$ $\; y \neq 0$ $\quad \sec\theta = \dfrac{r}{x}$ $\; x \neq 0$ $\quad \cot\theta = \dfrac{x}{y}$ $\; y \neq 0$

Periodic function (p. 409) — $f(\theta + p) = f(\theta)$, for all θ, $p > 0$, where the smallest such p is the fundamental period