6.7 Assess Your Understanding

Skill Building

In Problems 1–6, find the exact value of each expression.

1. $\sin 195° \cdot \cos 75°$ **2.** $\cos 285° \cdot \cos 195°$ **3.** $\sin 285° \cdot \sin 75°$

4. $\sin 75° + \sin 15°$ **5.** $\cos 255° - \cos 195°$ **6.** $\sin 255° - \sin 15°$

In Problems 7–16, express each product as a sum containing only sines or only cosines.

7. $\sin(4\theta)\sin(2\theta)$ **8.** $\cos(4\theta)\cos(2\theta)$ **9.** $\sin(4\theta)\cos(2\theta)$ **10.** $\sin(3\theta)\sin(5\theta)$ **11.** $\cos(3\theta)\cos(5\theta)$

12. $\sin(4\theta)\cos(6\theta)$ **13.** $\sin\theta\sin(2\theta)$ **14.** $\cos(3\theta)\cos(4\theta)$ **15.** $\sin\frac{3\theta}{2}\cos\frac{\theta}{2}$ **16.** $\sin\frac{\theta}{2}\cos\frac{5\theta}{2}$

In Problems 17–24, express each sum or difference as a product of sines and/or cosines.

17. $\sin(4\theta) - \sin(2\theta)$ **18.** $\sin(4\theta) + \sin(2\theta)$ **19.** $\cos(2\theta) + \cos(4\theta)$ **20.** $\cos(5\theta) - \cos(3\theta)$

21. $\sin\theta + \sin(3\theta)$ **22.** $\cos\theta + \cos(3\theta)$ **23.** $\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}$ **24.** $\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}$

In Problems 25–42, establish each identity.

25. $\dfrac{\sin\theta + \sin(3\theta)}{2\sin(2\theta)} = \cos\theta$ **26.** $\dfrac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \cos\theta$ **27.** $\dfrac{\sin(4\theta) + \sin(2\theta)}{\cos(4\theta) + \cos(2\theta)} = \tan(3\theta)$

28. $\dfrac{\cos\theta - \cos(3\theta)}{\sin(3\theta) - \sin\theta} = \tan(2\theta)$ **29.** $\dfrac{\cos\theta - \cos(3\theta)}{\sin\theta + \sin(3\theta)} = \tan\theta$ **30.** $\dfrac{\cos\theta - \cos(5\theta)}{\sin\theta + \sin(5\theta)} = \tan(2\theta)$

31. $\sin\theta[\sin\theta + \sin(3\theta)] = \cos\theta[\cos\theta - \cos(3\theta)]$ **32.** $\sin\theta[\sin(3\theta) + \sin(5\theta)] = \cos\theta[\cos(3\theta) - \cos(5\theta)]$

33. $\dfrac{\sin(4\theta) + \sin(8\theta)}{\cos(4\theta) + \cos(8\theta)} = \tan(6\theta)$ **34.** $\dfrac{\sin(4\theta) - \sin(8\theta)}{\cos(4\theta) - \cos(8\theta)} = -\cot(6\theta)$

35. $\dfrac{\sin(4\theta) + \sin(8\theta)}{\sin(4\theta) - \sin(8\theta)} = -\dfrac{\tan(6\theta)}{\tan(2\theta)}$ **36.** $\dfrac{\cos(4\theta) - \cos(8\theta)}{\cos(4\theta) + \cos(8\theta)} = \tan(2\theta)\tan(6\theta)$

37. $\dfrac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \tan\dfrac{\alpha+\beta}{2}\cot\dfrac{\alpha-\beta}{2}$ **38.** $\dfrac{\cos\alpha + \cos\beta}{\cos\alpha - \cos\beta} = -\cot\dfrac{\alpha+\beta}{2}\cot\dfrac{\alpha-\beta}{2}$

39. $\dfrac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \tan\dfrac{\alpha+\beta}{2}$ **40.** $\dfrac{\sin\alpha - \sin\beta}{\cos\alpha - \cos\beta} = -\cot\dfrac{\alpha+\beta}{2}$

41. $1 + \cos(2\theta) + \cos(4\theta) + \cos(6\theta) = 4\cos\theta\cos(2\theta)\cos(3\theta)$

42. $1 - \cos(2\theta) + \cos(4\theta) - \cos(6\theta) = 4\sin\theta\cos(2\theta)\sin(3\theta)$

In Problems 43–46, solve each equation on the interval $0 \le \theta < 2\pi$.

43. $\sin(2\theta) + \sin(4\theta) = 0$ **44.** $\cos(2\theta) + \cos(4\theta) = 0$

45. $\cos(4\theta) - \cos(6\theta) = 0$ **46.** $\sin(4\theta) - \sin(6\theta) = 0$

Applications and Extensions

47. Touch-Tone Phones On a Touch-Tone phone, each button produces a unique sound. The sound produced is the sum of two tones, given by

$$y = \sin(2\pi l t) \quad \text{and} \quad y = \sin(2\pi h t)$$

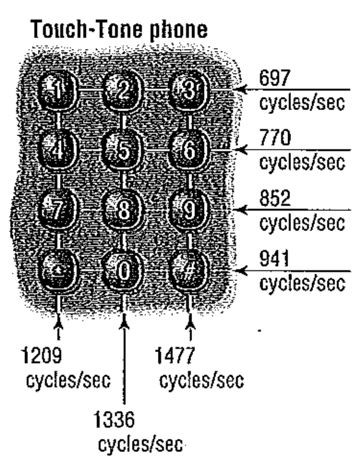
where l and h are the low and high frequencies (cycles per second) shown on the illustration. For example, if you touch 1, the low frequency is $l = 697$ cycles per second and the high frequency is $h = 1209$ cycles per second. The sound emitted by touching 1 is

$$y = \sin[2\pi(697)t] + \sin[2\pi(1209)t]$$

(a) Write this sound as a product of sines and/or cosines.

(b) Determine the maximum value of y.

(c) Graph the sound emitted by touching 1.



48. **Touch-Tone Phones**
(a) Write the sound emitted by touching the # key as a product of sines and/or cosines.
(b) Determine the maximum value of y.
(c) Graph the sound emitted by touching the # key.

49. **Moment of Inertia** The moment of inertia I of an object is a measure of how easy it is to rotate the object about some fixed point. In engineering mechanics, it is sometimes necessary to compute moments of inertia with respect to a set of rotated axes. These moments are given by the equations

$$I_u = I_x \cos^2\theta + I_y \sin^2\theta - 2I_{xy}\sin\theta\cos\theta$$

$$I_v = I_x \sin^2\theta + I_y \cos^2\theta + 2I_{xy}\sin\theta\cos\theta$$

Use Product-to-Sum Formulas to show that

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos(2\theta) - I_{xy}\sin(2\theta)$$

and

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos(2\theta) + I_{xy}\sin(2\theta)$$

Source: Adapted from Hibbeler, *Engineering Mechanics: Statics*, 10th ed., Prentice Hall © 2004.

50. **Projectile Motion** The range R of a projectile propelled downward from the top of an inclined plane at an angle θ to the inclined plane is given by

$$R(\theta) = \frac{2v_0^2 \sin\theta\cos(\theta - \phi)}{g\cos^2\phi}$$

where v_0 is the initial velocity of the projectile, ϕ is the angle the plane makes with respect to the horizontal, and g is acceleration due to gravity.
(a) Show that for fixed v_0 and ϕ, the maximum range down the incline is given by $R_{\max} = \dfrac{v_0^2}{g(1 - \sin\phi)}$.
(b) Determine the maximum range if the projectile has an initial velocity of 50 meters/second, the angle of the plane is $\phi = 35°$, and $g = 9.8$ meters/second^2.

51. If $\alpha + \beta + \gamma = \pi$, show that

$$\sin(2\alpha) + \sin(2\beta) + \sin(2\gamma) = 4\sin\alpha\sin\beta\sin\gamma$$

52. If $\alpha + \beta + \gamma = \pi$, show that

$$\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha\tan\beta\tan\gamma$$

53. Derive formula (3).
54. Derive formula (7).
55. Derive formula (8).
56. Derive formula (9).

Retain Your Knowledge

Problems 57–60 are based on material learned earlier in the course. The purpose of these problems is to keep the material fresh in your mind so that you are better prepared for the final exam.

57. Solve: $27^{x+1} = 9^{x+5}$
58. For $y = 5\cos(4x - \pi)$, find the amplitude, the period, and the phase shift.
59. Find the exact value of $\cos\left(\csc^{-1}\frac{7}{5}\right)$.
60. Find the inverse function f^{-1} of $f(x) = 3\sin x - 5$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Find the range of f and the domain and range of f^{-1}.

Chapter Review

Things to Know

Definitions of the six inverse trigonometric functions

$y = \sin^{-1} x$ means $x = \sin y$ where $-1 \le x \le 1$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (p. 465)

$y = \cos^{-1} x$ means $x = \cos y$ where $-1 \le x \le 1$, $0 \le y \le \pi$ (p. 468)

$y = \tan^{-1} x$ means $x = \tan y$ where $-\infty < x < \infty$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (p. 471)

$y = \sec^{-1} x$ means $x = \sec y$ where $|x| \ge 1$, $0 \le y \le \pi$, $y \ne \frac{\pi}{2}$ (p. 478)

$y = \csc^{-1} x$ means $x = \csc y$ where $|x| \ge 1$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$ (p. 478)

$y = \cot^{-1} x$ means $x = \cot y$ where $-\infty < x < \infty$, $0 < y < \pi$ (p. 478)