

Skill Building

In Problems 7–16, tell whether the expression is a monomial. If it is, name the variable(s) and the coefficient and give the degree of the monomial. If it is not a monomial, state why not.

7. $2x^3$ 8. $-4x^2$ 9. $\frac{8}{x}$ 10. $-2x^{-3}$ 11. $-2xy^2$
 12. $5x^2y^3$ 13. $\frac{8x}{y}$ 14. $-\frac{2x^2}{y^3}$ 15. $x^2 + y^2$ 16. $3x^2 + 4$

In Problems 17–26, tell whether the expression is a polynomial. If it is, give its degree. If it is not, state why not.

17. $3x^2 - 5$ 18. $1 - 4x$ 19. 5 20. $-\pi$ 21. $3x^2 - \frac{5}{x}$
 22. $\frac{3}{x} + 2$ 23. $2y^3 - \sqrt{2}$ 24. $10z^2 + z$ 25. $\frac{x^2 + 5}{x^3 - 1}$ 26. $\frac{3x^3 + 2x - 1}{x^2 + x + 1}$

In Problems 27–46, add, subtract, or multiply, as indicated. Express your answer as a single polynomial in standard form.

27. $(x^2 + 4x + 5) + (3x - 3)$ 28. $(x^3 + 3x^2 + 2) + (x^2 - 4x + 4)$
 29. $(x^3 - 2x^2 + 5x + 10) - (2x^2 - 4x + 3)$ 30. $(x^2 - 3x - 4) - (x^3 - 3x^2 + x + 5)$
 31. $(6x^5 + x^3 + x) + (5x^4 - x^3 + 3x^2)$ 32. $(10x^5 - 8x^2) + (3x^3 - 2x^2 + 6)$
 33. $(x^2 - 3x + 1) + 2(3x^2 + x - 4)$ 34. $-2(x^2 + x + 1) + (-5x^2 - x + 2)$
 35. $6(x^3 + x^2 - 3) - 4(2x^3 - 3x^2)$ 36. $8(4x^3 - 3x^2 - 1) - 6(4x^3 + 8x - 2)$
 37. $(x^2 - x + 2) + (2x^2 - 3x + 5) - (x^2 + 1)$ 38. $(x^2 + 1) - (4x^2 + 5) + (x^2 + x - 2)$
 39. $9(y^2 - 3y + 4) - 6(1 - y^2)$ 40. $8(1 - y^3) + 4(1 + y + y^2 + y^3)$
 41. $x(x^2 + x - 4)$ 42. $4x^2(x^3 - x + 2)$
 43. $-2x^2(4x^3 + 5)$ 44. $5x^3(3x - 4)$
 45. $(x + 1)(x^2 + 2x - 4)$ 46. $(2x - 3)(x^2 + x + 1)$

In Problems 47–64, multiply the polynomials using the FOIL method. Express your answer as a single polynomial in standard form.

47. $(x + 2)(x + 4)$ 48. $(x + 3)(x + 5)$ 49. $(2x + 5)(x + 2)$
 50. $(3x + 1)(2x + 1)$ 51. $(x - 4)(x + 2)$ 52. $(x + 4)(x - 2)$
 53. $(x - 3)(x - 2)$ 54. $(x - 5)(x - 1)$ 55. $(2x + 3)(x - 2)$
 56. $(2x - 4)(3x + 1)$ 57. $(-2x + 3)(x - 4)$ 58. $(-3x - 1)(x + 1)$
 59. $(-x - 2)(-2x - 4)$ 60. $(-2x - 3)(3 - x)$ 61. $(x - 2y)(x + y)$
 62. $(2x + 3y)(x - y)$ 63. $(-2x - 3y)(3x + 2y)$ 64. $(x - 3y)(-2x + y)$

In Problems 65–88, multiply the polynomials using the special product formulas. Express your answer as a single polynomial in standard form.

65. $(x - 7)(x + 7)$ 66. $(x - 1)(x + 1)$ 67. $(2x + 3)(2x - 3)$
 68. $(3x + 2)(3x - 2)$ 69. $(x + 4)^2$ 70. $(x + 5)^2$
 71. $(x - 4)^2$ 72. $(x - 5)^2$ 73. $(3x + 4)(3x - 4)$
 74. $(5x - 3)(5x + 3)$ 75. $(2x - 3)^2$ 76. $(3x - 4)^2$
 77. $(x + y)(x - y)$ 78. $(x + 3y)(x - 3y)$ 79. $(3x + y)(3x - y)$
 80. $(3x + 4y)(3x - 4y)$ 81. $(x + y)^2$ 82. $(x - y)^2$
 83. $(x - 2y)^2$ 84. $(2x + 3y)^2$ 85. $(x - 2)^3$
 86. $(x + 1)^3$ 87. $(2x + 1)^3$ 88. $(3x - 2)^3$

In Problems 89–104, find the quotient and the remainder. Check your work by verifying that

$$(\text{Quotient})(\text{Divisor}) + \text{Remainder} = \text{Dividend}$$

89. $4x^3 - 3x^2 + x + 1$ divided by $x + 2$ 90. $3x^3 - x^2 + x - 2$ divided by $x + 2$
 91. $4x^3 - 3x^2 + x + 1$ divided by x^2 92. $3x^3 - x^2 + x - 2$ divided by x^2
 93. $5x^4 - 3x^2 + x + 1$ divided by $x^2 + 2$ 94. $5x^4 - x^2 + x - 2$ divided by $x^2 + 2$
 95. $4x^5 - 3x^2 + x + 1$ divided by $2x^3 - 1$ 96. $3x^5 - x^2 + x - 2$ divided by $3x^3 - 1$
 97. $2x^4 - 3x^3 + x + 1$ divided by $2x^2 + x + 1$ 98. $3x^4 - x^3 + x - 2$ divided by $3x^2 + x + 1$
 99. $-4x^3 + x^2 - 4$ divided by $x - 1$ 100. $-3x^4 - 2x - 1$ divided by $x - 1$
 101. $1 - x^2 + x^4$ divided by $x^2 + x + 1$ 102. $1 - x^2 + x^4$ divided by $x^2 - x + 1$
 103. $x^3 - a^3$ divided by $x - a$ 104. $x^5 - a^5$ divided by $x - a$

In Problems 15–22, factor the difference of two squares.

15. $x^2 - 1$ 16. $x^2 - 4$ 17. $4x^2 - 1$ 18. $9x^2 - 1$
 19. $x^2 - 16$ 20. $x^2 - 25$ 21. $25x^2 - 4$ 22. $36x^2 - 9$

In Problems 23–32, factor the perfect squares.

23. $x^2 + 2x + 1$ 24. $x^2 - 4x + 4$ 25. $x^2 + 4x + 4$ 26. $x^2 - 2x + 1$
 27. $x^2 - 10x + 25$ 28. $x^2 + 10x + 25$ 29. $4x^2 + 4x + 1$ 30. $9x^2 + 6x + 1$
 31. $16x^2 + 8x + 1$ 32. $25x^2 + 10x + 1$

In Problems 33–38, factor the sum or difference of two cubes.

33. $x^3 - 27$ 34. $x^3 + 125$ 35. $x^3 + 27$ 36. $27 - 8x^3$ 37. $8x^3 + 27$ 38. $64 - 27x^3$

In Problems 39–50, factor each polynomial.

39. $x^2 + 5x + 6$ 40. $x^2 + 6x + 8$ 41. $x^2 + 7x + 6$
 42. $x^2 + 9x + 8$ 43. $x^2 + 7x + 10$ 44. $x^2 + 11x + 10$
 45. $x^2 - 10x + 16$ 46. $x^2 - 17x + 16$ 47. $x^2 - 7x - 8$
 48. $x^2 - 2x - 8$ 49. $x^2 + 7x - 8$ 50. $x^2 + 2x - 8$

In Problems 51–56, factor by grouping.

51. $2x^2 + 4x + 3x + 6$ 52. $3x^2 - 3x + 2x - 2$
 53. $2x^2 - 4x + x - 2$ 54. $3x^2 + 6x - x - 2$
 55. $6x^2 + 9x + 4x + 6$ 56. $9x^2 - 6x + 3x - 2$

In Problems 57–68, factor each polynomial.

57. $3x^2 + 4x + 1$ 58. $2x^2 + 3x + 1$ 59. $2z^2 + 5z + 3$ 60. $6z^2 + 5z + 1$
 61. $3x^2 + 2x - 8$ 62. $3x^2 + 10x + 8$ 63. $3x^2 - 2x - 8$ 64. $3x^2 - 10x + 8$
 65. $3x^2 + 14x + 8$ 66. $3x^2 - 14x + 8$ 67. $3x^2 + 10x - 8$ 68. $3x^2 - 10x - 8$

In Problems 69–74, determine the number that should be added to complete the square of each expression. Then factor each expression.

69. $x^2 + 10x$ 70. $p^2 + 14p$ 71. $y^2 - 6y$
 72. $x^2 - 4x$ 73. $x^2 - \frac{1}{2}x$ 74. $x^2 + \frac{1}{3}x$

Mixed Practice

In Problems 75–122, factor each polynomial completely. If the polynomial cannot be factored, say it is prime.

75. $x^2 - 36$ 76. $x^2 - 9$ 77. $2 - 8x^2$ 78. $3 - 27x^2$
 79. $x^2 + 11x + 10$ 80. $x^2 + 5x + 4$ 81. $x^2 - 10x + 21$ 82. $x^2 - 6x + 8$
 83. $4x^2 - 8x + 32$ 84. $3x^2 - 12x + 15$ 85. $x^2 + 4x + 16$ 86. $x^2 + 12x + 36$
 87. $15 + 2x - x^2$ 88. $14 + 6x - x^2$ 89. $3x^2 - 12x - 36$ 90. $x^3 + 8x^2 - 20x$
 91. $y^4 + 11y^3 + 30y^2$ 92. $3y^3 - 18y^2 - 48y$ 93. $4x^2 + 12x + 9$ 94. $9x^2 - 12x + 4$
 95. $6x^2 + 8x + 2$ 96. $8x^2 + 6x - 2$ 97. $x^4 - 81$ 98. $x^4 - 1$
 99. $x^6 - 2x^3 + 1$ 100. $x^6 + 2x^3 + 1$ 101. $x^7 - x^5$ 102. $x^8 - x^5$
 103. $16x^2 + 24x + 9$ 104. $9x^2 - 24x + 16$ 105. $5 + 16x - 16x^2$ 106. $5 + 11x - 16x^2$
 107. $4y^2 - 16y + 15$ 108. $9y^2 + 9y - 4$ 109. $1 - 8x^2 - 9x^4$ 110. $4 - 14x^2 - 8x^4$
 111. $x(x + 3) - 6(x + 3)$ 112. $5(3x - 7) + x(3x - 7)$ 113. $(x + 2)^2 - 5(x + 2)$
 114. $(x - 1)^2 - 2(x - 1)$ 115. $(3x - 2)^3 - 27$ 116. $(5x + 1)^3 - 1$
 117. $3(x^2 + 10x + 25) - 4(x + 5)$ 118. $7(x^2 - 6x + 9) + 5(x - 3)$ 119. $x^3 + 2x^2 - x - 2$
 120. $x^3 - 3x^2 - x + 3$ 121. $x^4 - x^3 + x - 1$ 122. $x^4 + x^3 + x + 1$

Applications and Extensions

In Problems 123–132, expressions that occur in calculus are given. Factor each expression completely.

123. $2(3x + 4)^2 + (2x + 3) \cdot 2(3x + 4) \cdot 3$ 124. $5(2x + 1)^2 + (5x - 6) \cdot 2(2x + 1) \cdot 2$
 125. $2x(2x + 5) + x^2 \cdot 2$ 126. $3x^2(8x - 3) + x^3 \cdot 8$
 127. $2(x + 3)(x - 2)^3 + (x + 3)^2 \cdot 3(x - 2)^2$ 128. $4(x + 5)^3(x - 1)^2 + (x + 5)^4 \cdot 2(x - 1)$